New Methods For Approximating Acoustic Wave Transmission Through Ducts

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August 21, 2007

Abstract

[illegible]

Con en

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Con o p o of φ o d c fo fo pp d n

f p o of φ o d c fo fo pp d n

p o of u o d c fo fo pp d n

Chapter 1

Introduction

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n C p 2 [illegible] d n od c d o c n p o o n n n d o n y o p o on n p d o nco n nd o o n d co d nc c n p o d [d o n on pp o d o c od [d d op on nd d op on o o n d on o on d c on o n pp o on oo fo y n d od n n o cco n p on f o co c n on o d c A of f o nc d od pp o on fo pp d p o nd d c n c of N nn o nd y cond on on d c

Chap er

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n n n o p op n o y n

dc n n n p

nd $\mathbf{k}_n$ po oo of

$$\boldsymbol{\sigma} \quad -\mathbf{k}_n \quad n\,\mathbf{k}_n\mathbf{h},$$

n d o $\mathbf{k}_n \leq \mathbf{k}_n$ $n \geq$ co c n n nd d

$$\mathbf{c} \quad \mathbf{c}\ \mathbf{h} \quad \sqrt{\mathbf{k}\ \mathbf{kh} \quad \mathbf{kh}^{-}},$$

$$\mathbf{c}_n \quad \mathbf{c}_n\ \mathbf{h} \quad \sqrt{\mathbf{k}_n\ \mathbf{k}_n\mathbf{h} \quad \mathbf{k}_n\mathbf{h}^{-}}, \quad n \geq \ ,$$

dc n n n p

No p on [d n **h** con n cond p on o o p fy on (2.2) o d op on

$$\nabla_{\mathbf{h}}.\mathbf{u} \quad \nabla_{\mathbf{h}}\boldsymbol{\varphi} \quad \mathbf{u} \; \mathbf{k}^{2}\boldsymbol{\varphi} \qquad (2.2)$$

dc n n n p

op on

M Mode Appro on

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o d n $\mathbf{v}$ no on $\mathbf{v}^{(j)}$ o co pond $\mathbf{a_j}$ nd $\mathbf{b_j}$ $\mathbf{j}$,

o o n $\mathbf{v}^{(j)}$ cond on d n d y o nd y cond on n y $\mathbf{Y}$ $\mathbf{b_j}$ nc

$$\widetilde{\mathbf{A}} \quad \mathrm{n} \Big[\surd$$

oc y po n oc y o o o d
y p on of d op on d c d n
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nd ne e on

n p on n o con d on o

n c n o

$$\int_a^b [u\,Y\ xx\ \ u\,Y\ yy\ \ k^2 u\,Y\,]\,Y\ dy$$

$$\int_a^b [u\ {}_{xx}Y\ \ 2\ Y\ {}_x u\ {}_x\ \ u\ Y\ {}_{xx}\ \ Y\ {}_{yy}\ \ k^2 Y\,]\,Y\ dy$$

$$\alpha u\ \ \alpha u\ \ \beta\ \ \gamma\ \ k^2\alpha\ u$$

$$\alpha u\ \ \beta\ \ \gamma\ \ k^2\alpha\ u$$

d no d n on p c o x

$$\alpha\ \ Y\ , Y\ ,\qquad \beta\ \ Y\ , Y\ ,\qquad \gamma\ \ Y$$

No $\boldsymbol{\beta}$ n $\mathbf{a}'$ $\mathbf{b}'$ $\mathbf{a}''$ $\mathbf{b}''$

fo d p od c

$$\mathbf{u}$$ $$\mathbf{R},$$

$$\mathbf{u}' \quad \mathbf{i}\kappa^{(\,)} - \mathbf{R}\,,$$

$$\Rightarrow \mathbf{u}' \quad \mathbf{i}\kappa^{(\,)}\mathbf{u} \quad \mathbf{i}\kappa^{(\,)} - \mathbf{R} \quad \mathbf{i}\kappa^{(\,)} \quad \mathbf{R}$$

$$\Rightarrow \quad \mathbf{u}' \quad \mathbf{i}\kappa^{(\,)}\mathbf{u} \quad \mathbf{i}\kappa^{(\,)}.$$

nd o nd y cond on fo nd n on n n $\mathbf{x}$ o

$$\mathbf{u}' - \mathbf{i}\kappa^{(\,)}\mathbf{u}\,.$$

G c n e e oc y Po en O er e o e D c

o c c φ n nd n on [o on nd o nd y cond on don n M n p o bvp4c o o nd y p o fo n c o nd y n od nd co p n n on n no pp op fo cc c no fo p o y p n n o on

n n bvp4c

De er n n e Bo nd ry Cond on

A n p o o d n o nd y cond on $\mathbf{x}$ nd $\mathbf{x}$ on of d c o n d oo y o po n n p nd d n con n o $\mathbf{x}$ fo

$$\mathbf{u}^{-} \quad \mathbf{u} \quad ,$$
$$\mathbf{u}'^{-} \quad \mathbf{u}' \quad ,$$

$\mathbf{u}^{\pm}$ d no $\mathbf{u}$ y o f o of z o cco d n y nd y fo $\mathbf{u}^{\pm}$

fo p o c on o d n $\mathbf{u}^{-}$ nd $\mathbf{u}'^{-}$ nd n co n n nn n n no n $\mathbf{A}$

$$\mathbf{u}^{-} \quad \mathbf{A},$$
$$\mathbf{u}'^{-} \quad \mathbf{A}\sqrt{\boldsymbol{\nu}^{(\ )} - \mathbf{k}^2},$$
$$\Rightarrow \mathbf{u}' - \sqrt{\boldsymbol{\nu}^{(\ )} - \mathbf{k}^2}\mathbf{u} \quad \mathbf{A}\sqrt{\boldsymbol{\nu}^{(\ )} - \mathbf{k}^2} - \sqrt{\boldsymbol{\nu}^{(\ )} - \mathbf{k}^2}\mathbf{A}$$
$$\Rightarrow \mathbf{u}' - \sqrt{\boldsymbol{\nu}^{(\ )} - \mathbf{k}^2}\mathbf{u} \quad .$$

n pp o fo $\mathbf{x}$

$$\mathbf{u}' \quad \sqrt{\boldsymbol{\nu}^{(\ )} - \mathbf{k}^2}\mathbf{u}$$

o n e E en e Pro e

y o d n on d n n p o n

oo of fnc on f ny n

oo of plo c n fo nd n c on od e oo [nd n o o y p dyd dn n n nf nd n c n n n e oo o o

no

$$\mathbf{Y_n x, y} \quad n\left(\frac{\mathbf{n\pi\, y - a}}{\mathbf{b - a}}\right) \qquad \mathbf{n} \quad \mathbf{, \ldots N.}$$

nc c nno p c ∇^2 $\mathbf{k}^2$ $\widetilde{\boldsymbol{\varphi}}$ fo of o z on n

$$\int_{\mathbf{a}}^{\mathbf{b}} \nabla$$

No α

nd

$$\sum_{n}^{N} i\kappa_n^{( )} I_n - R_n \ Y_n y \quad \sum_{n}^{N} u_n' \ Y_n , y$$

$$\Rightarrow i\kappa_n^{( )} I_n - R_n \quad u_n' \ , \qquad n \ , .., N.$$

pon n n n no n R_n fo o on o nd y cond on

$$u_n' \quad i\kappa_n^{( )} u_n \quad i\kappa_n^{( )} I_n \qquad n \ , .., N.$$

o o n p oc d c n nd nd o nd y cond on n $x >$

$$\varphi \approx \sum_{n}^{N} T_n e^{i\kappa_n^{(1)} x} Y_n y \ .$$

T_n n no n co c n of n^{th} n d od n $< x <$ fo

$$\varphi \approx \sum_{n}^{N} u_n x \ Y_n x, y \ .$$

A p o c fo con n y o on nd d n f c x

$$\sum_{n}^{N} T_n e^{i\kappa_n^{(1)}} Y_n y \quad \sum_{n}^{N} u_n \ Y_n , y$$

$$\Rightarrow T_n e^{i\kappa_n^{(1)}} \quad u_n$$

nd

$$\sum_{n}^{N} i\kappa_n^{( )} T_n e^{i\kappa_n^{(1)}} Y_n y \quad \sum_{n}^{N} u_n' \ Y_n , y$$

$$\Rightarrow i\kappa_n^{( )} T_n e^{i\kappa_n^{(1)}} \quad u_n' \qquad n \ , .., N$$

n n T_n n no n fo on nd n o o cond o nd y cond on

$$u_n' - i\kappa_n^{( )} u_n \qquad n \ , .., N.$$

Chapter

N er e

e er n Pro e

A c n po d c d fy n y fo a,x nd b,x n n p nc of on nd on y on p op n

n e Mode Appro on

od n c o of d c od d [f d o d c c n d d y

$$
\mathbf{a}(x) = \begin{cases} & x < \\ \mathbf{d} \; \mathrm{n}^2\left(\frac{\pi x}{2}\right) & < x < \\ \mathbf{d} & x > , \end{cases}
$$

nd

$$
\mathbf{b}(x) = \begin{cases} & x < \\ -\mathbf{a}(x) & < x < \\ -\mathbf{d} & x > . \end{cases}
$$

nd o n d . k nd c n

o c y o f of d c n o

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n n n p o o c of no n

o op of d c fo $< x <$ o d c on

o c y no d p n con n

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[illegible] pond [illegible] $|\mathbf{R}|$ [illegible] $\mathbf{k}$ [illegible] pond [illegible] con [illegible] A [illegible] pon [illegible] conc[illegible] d [illegible] on [illegible] $\mathbf{k}$[illegible]

No [illegible] con [illegible] d [illegible]

$$\mathbf{a}(\mathbf{x}) = \begin{cases} & \mathbf{x} < \\ \mathbf{d}\ [illegible]\mathrm{n}^2\left(\frac{\mu\pi\mathbf{x}}{[illegible]}\right) & < \mathbf{x} < [illegible] \\ & \mathbf{x} > [illegible], \end{cases}$$

nd

$$\mathbf{b}(\mathbf{x}) = \begin{cases} & \mathbf{x} < \\ -\mathbf{a}(\mathbf{x}) & < \mathbf{x} < [illegible] \\ & \mathbf{x} > [illegible]. \end{cases}$$

[illegible] nd [illegible] $\mathbf{d}$ [illegible] $\mathbf{k}$ [illegible] nd μ [illegible] μ [illegible] $< \mathbf{x} <$ [illegible] conc[illegible] on [illegible]

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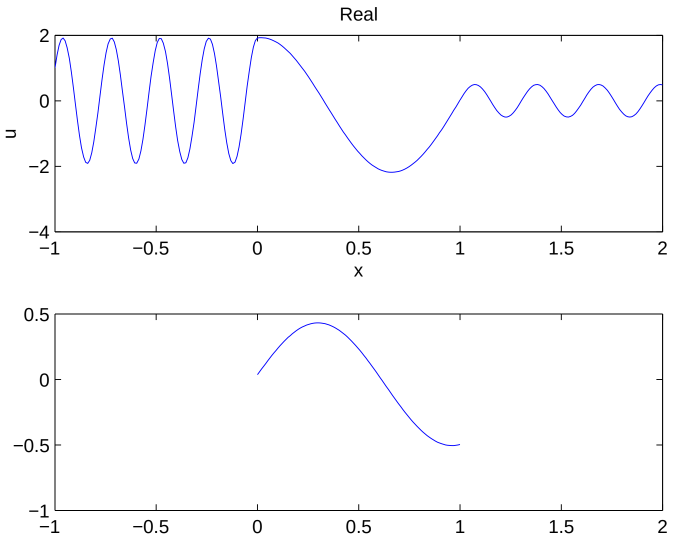
[illegible] $\mathbf{k}|\mathbf{R}|$ [illegible] $\mathbf{d}$ nd [illegible] μ [illegible] nd [illegible] μ [illegible] nd $\mathbf{d}$ [illegible] $\mathbf{k}|\mathbf{R}|$ [illegible] $\mathbf{k}|\mathbf{R}|$ [illegible] $\mathbf{d}$ [illegible] $\mathbf{d}$ [illegible] $\mathbf{k}|\mathbf{R}|$ occ[illegible] $\mathbf{d}$ [illegible] μ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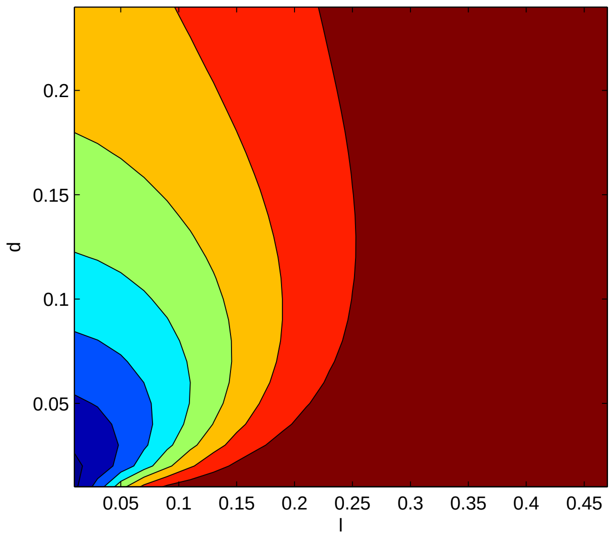
N c



0.2
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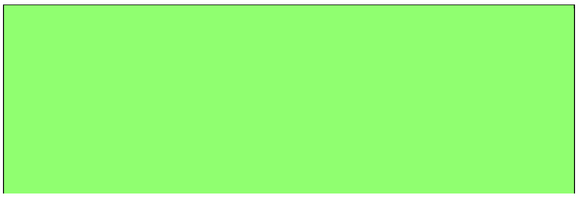
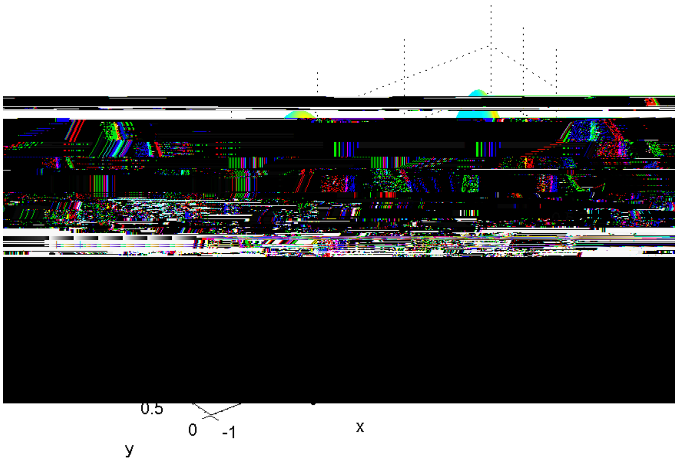
N c

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pped e Pro e

n e Mode Appro on $n = 1$

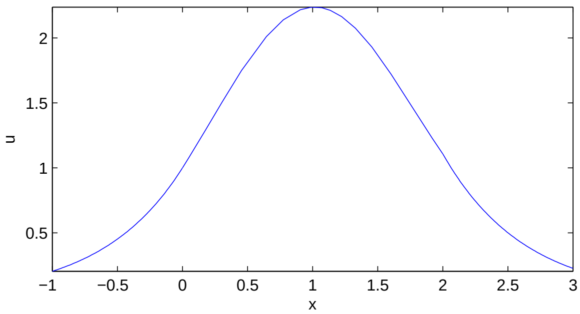
A n on d n d c on fo nd n oc y po n **φ** nd of **k** fo pp d nc n d c

$$\mathbf{a}(\mathbf{x}) = \begin{cases} & \mathbf{x} < \\ \mathbf{d}\ \mathrm{n}^2\left(\frac{\mu\pi\mathbf{x}}{}\right) & < \mathbf{x} < \\ & \mathbf{x} > , \end{cases}$$

nd

$$\mathbf{b}(\mathbf{x}) = \begin{cases} & \mathbf{x} < \\ -\mathbf{a}(\mathbf{x}) & < \mathbf{x} < \\ & \mathbf{x} > . \end{cases}$$

o fo p o oo **d** = . L n d c nd **μ** = o cond o nd y cond on p o d n **k** o p o d c o d n ny n fo od o zoo n on c on of p o o fy n n c on od oo fo nd o **k** = . of **k** n o p od c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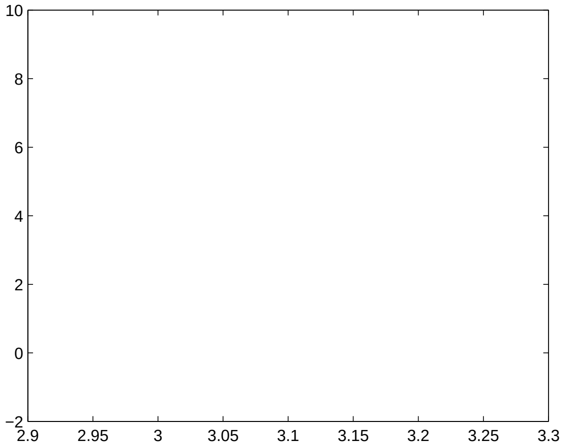
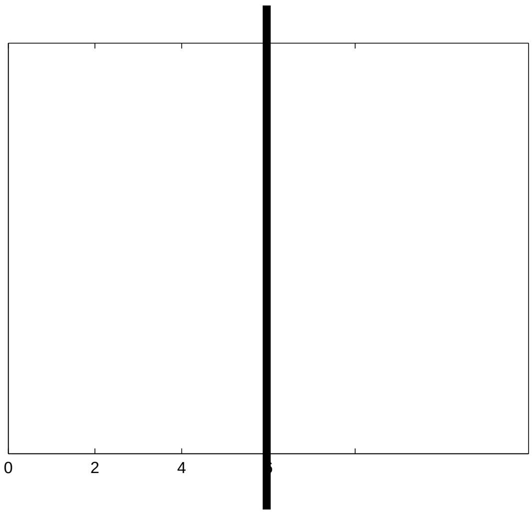
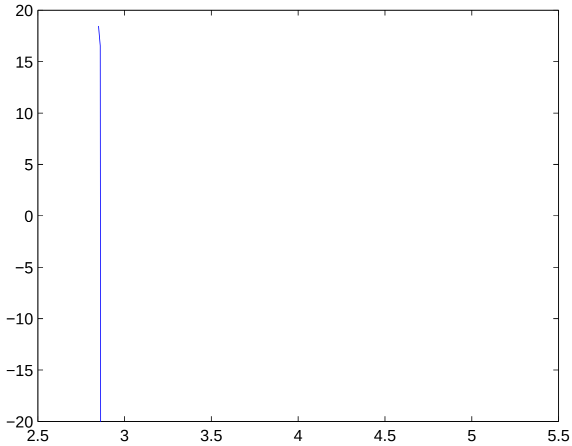


N c





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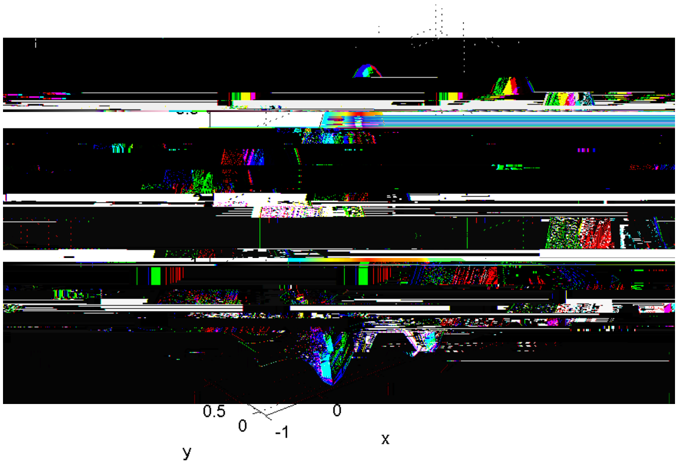
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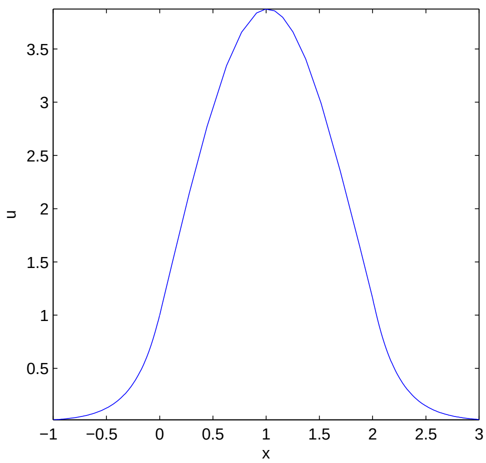
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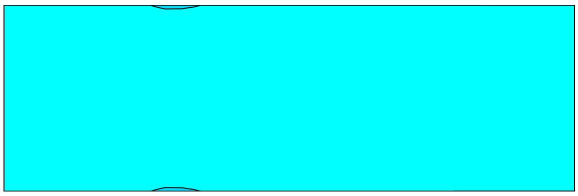

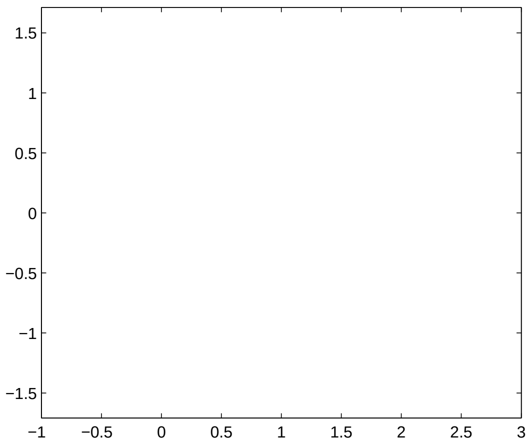
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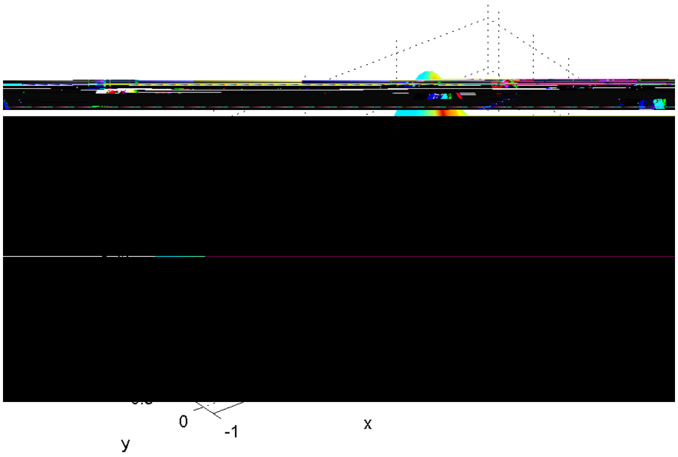
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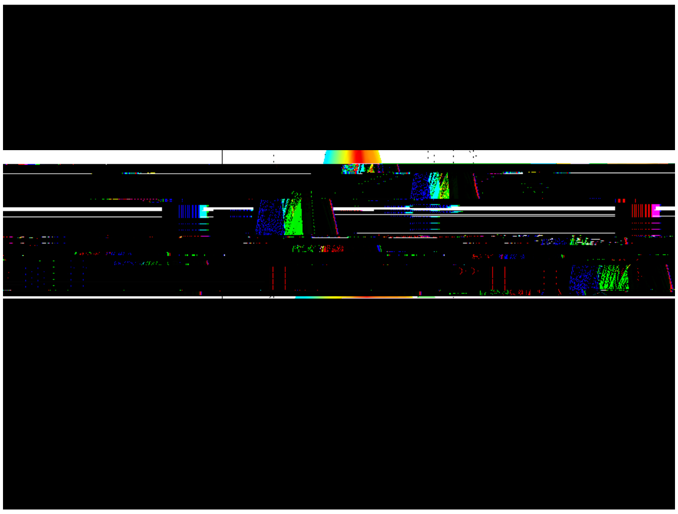
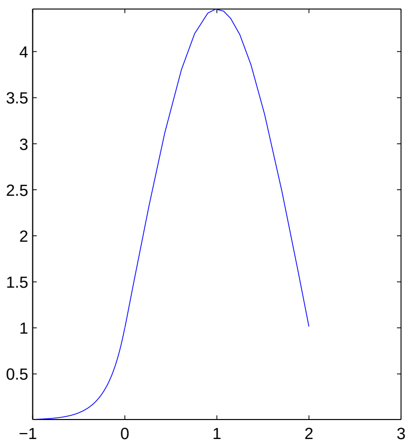


[illegible]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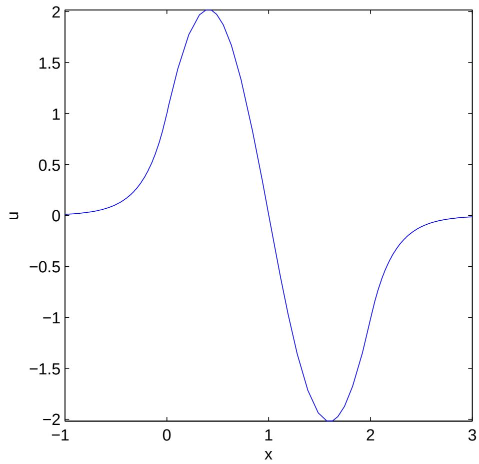
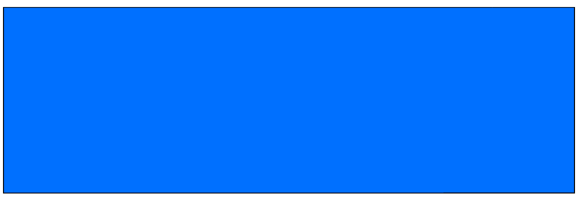
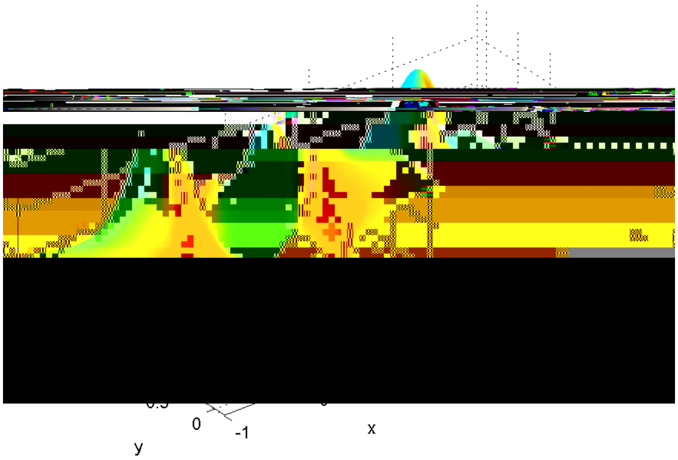


p o of u o d c for cond pp d

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on n o d n nn p o n od pp o on o n o [nd φ

O n od pp o on d d no on y p o c n p o o pp d p o p o n o d n d d on p f n o [nd u n n p o do [y d o d co on o d p o d pp d o [nd n pp o on o pp d p o no M p o d c d ode23 d n p on of o nd o nd y cond on o [nd n n o f p o n f c n p o

N n c of n on d od f od pp o on o n od pp o on nd c f c n p o cond on on d c n od pp o on c n A n od pp o on do no c of co c n o o d d [y o co c n of o pp o on o

2 O nd d z n on of d op on **J. Fluid Mechanics** **367-382**

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