Early Warning with Calibrated and Sharper Probabilistic Forecasts

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Abstract

Given a nonlinear deterministic model, a density forecast i

ar na ca brat on w n not a o at t sa t D ns t or asts ar sa to b probab st a a brat an on t I s ar un or str but arF na a brat on r rs to t as w n t t av raF o a ns t or asts s qua to t at o a or asts r s no p r a wa o ass ss nF x an a brat on

Gn t nF *ta* t n on tur t at w n a subs t o t s o o a brat on o s t n t pr t v str but ons ar at ast as spr a out as t a or asts w on tur t t r a *s arpn ss pr nc p* t r *s arpn ss* s s to av b n o n b Bross an t s a asur o ow on ntrat probab st or asts ar an a prop rt o o boF to t s attr but s w ato oF s u o s o a brat on ass ss ar probab st an arF na brat on w t p as s t at ntrop s ou b us to asur s arpn ss n w propos t on t at a r ss s Gn t nF a s on tur os xp a n t n nFs r as Bro r t s r t to not n u nF a n un t ons n t ns t st at on was ts Fuarant n r as n var an o t pr t v str but on o par to t raw ns b w onstrat t at t us o var an to asur s arpn ss ou b s a nF

s pap r s orFan s as o ows n xt s t on s uss s or ast qua t s t at ar u u at v asur b t oFar t s or nF ru In part u ar a o pos t on o t s s or nF ru s pr s nt s arpn ss pr n p on tur b Gn t nF a s s uss an a r vant propos t on pr s nt n § In § t t o o oF p o to pr u r st or asts s out n su ts on rn nF ns t or asts obta n v a t oFar t s or nF ru w t sp t to a non n ar tron r u t ar s uss n § t on F v s on u nF r ar s App n s A an B onta n t proo o t propos t on on rn nF t s arpn ss pr n p an app n x C onta ns proo s or t r st o t propos t ons

2 Probabilistic-Forecast Quality

o ssp at on p a s tat ons on t va u o probab st or asts n t ot r an onsu rs o r asts a an or t v str but ons t at ar bot *ca brat* an *s arp* I su or asts ar ssu at onF t or ons t n ar warn nF s a or suFF st t at t s qua t s an b u u at quant b t oFar t s or nF ru propos b Goo r ar ot r s or nF ru s ava ab or s t ons s Gn t nF a t r For nstan t r s t Br r s or Br r s owv r o pos s nto an t r s urp s o w ar not r vant to our s uss on an t s su tab or at r a v nts A F n ra sat on o t Br r s or to ns t or asts s t

o two Gauss ans w av ow r ntrop t a a s nF Gauss an str but on o t s var an u or a str but on o a F r var an an av a ow r ntrop t an t at o ow r var an

arp ss as a so b n quant b on n nt rva s a t r a Gn t nF a Con n nt rva s ar a s ar w a n ss to var an n t s ns t at a b o a str but on t at s a r on ntrat on t two o s an av arF r on n nt rva s t an a un o a str but on t at s a r spr a out A so F v n two non s tr str but ons w o t s s arp r ou p n on w at t on n v s

2.3 Calibration

a brat on o ns t or asts s a w tro n sub t u o t t ratur ta s t stan t at a a brat or ast nF s st tanta ount to a orr t sp Corra wanson prov a o pr ns v surv o o a stat st a t n qu s or ass ss nF a brat on o ns t or asts to t n t un r nF o s orr t so wor o Gn t nF a str s a s or b prov nF a a brat on ra wor t at a o at s o ssp at on bro own a brat on nto t r og s a o w ou b ass ss s parat uppos a probab t or ast nF s st ssu s pr t v str but ons $\{ _t x \}_{t=1}^{T}$ w t ata F n rat nF pro ss ssu s a or asts $\{ _t x \}_{t=1}^{T}$ Gn t nF a t n n t o ow nF o s o a brat on

- s qu n $_t x \}_{t=1}^{T}$ s *probab s a y ca brat* r at v to $\{ _t x \}_{t=1}^{T}$

$$\sum_{t=1}^{T} {}_t\{ {}_t^{-1} p \} \quad p \quad p$$

- s qu n $\{ _t x \}_{t=1}^{T}$ s *xc an ca brat* r at v t $\{ _t x \}_{t=1}^{T}$

$$\sum_{t=1}^{T} {}_t^{-1}\{ {}_t x \} \quad x \quad x$$

- or ast r s *ar na y ca brat*

$$\frac{}{T}\sum_{t=1}^{T} {}_t x \qquad \frac{}{T}\sum_{t=1}^{T} {}_t x$$

I w av a t s r s o obs rvat ons t n $_t$ $_t x_t$ s a *probab y nt a trans or* I Corra wanson D o a n o t o t I s s u a nt to probab st a brat on Gn t nF a A v sua nsp t on o I sto ra s wou r v a obv ous partur s ro un or t un r nF o s orr t sp an on $_t$

uppos w av a t m s r s o ns t or asts $\{ _t x \}_{t\ 1}$ n n t or ast r s m ato oF as

$$_T x \quad \sum_{t=1}^{T} \ _t x$$

n a or ast r w o ssu s

$$_t x \quad \overline{\ \ }_T x \quad \sum_{t=1}^{T} \ _t x$$

or a { } to b t *n t c ato o ca or cast* I $_t x$ m $_T$ $\overline{\ }_T x$ t n w av t *c ato o ca or cast* A or ast r s *n t ar na y ca brat* $_T x$ $\overline{\ }_T x$

For a pra t a purpos s s n t an w av no a ss to t $_t x$ s H n t s u t to ass ss n t m arF na a brat on I $_T$ $_T$ ≈ w r s s m tr t n w an ta to b arF nouF to va uat m arF na a brat on o t s n w an us t H nF r stan o ar an o m put

$$_T \ _c$$

Gndddd.6

w an or ns t or ast st at s o t or

$$ {}^{(t)}\ x \quad \frac{}{}\sum_{i=1}^{N} \left\{ \left(x - X_i^{(t)} - \right. \right\} $$

w r an ar r sp t v ban w t an o s t para t rs os n a or nF to past r or an an s t rn un t on ns t or ast n rs ro t tra t ona ar n st at s b t o s t para t r It s s ar to t Ba s an o Av raF propos b a t r a w a m or b as orr t on an qua w F ts H r t ns b b rs ar x anF ab an o not r pr s nt st n t o s t nF us nF v r an o s not a ount or o ssp at on

o a ount or o ssp at on t us rst not a r or o past t s r s an rr spon nF ns b or asts b V_T $\{ \ _t\ X^{(t)} \}_{t=1}^{T}$ n t ns t or asts w os para t rs an a s t b a nF nto a ount past p r or an a b not b $^{(t)}\ x|V_T$ $^{(t)}\ x|V_T$ as t sa or as n ts para t rs ar s t b o nF t n sat on

$$ \min_{\sigma>0,\mu} \quad \frac{}{} \sum_{t=1}^{T} \ \mathrm{oF}\ {}^{(t)} \quad {}_t|V_T $$

v n r rta n assu pt ons o nF t n sat on n s an t para t rs c an c

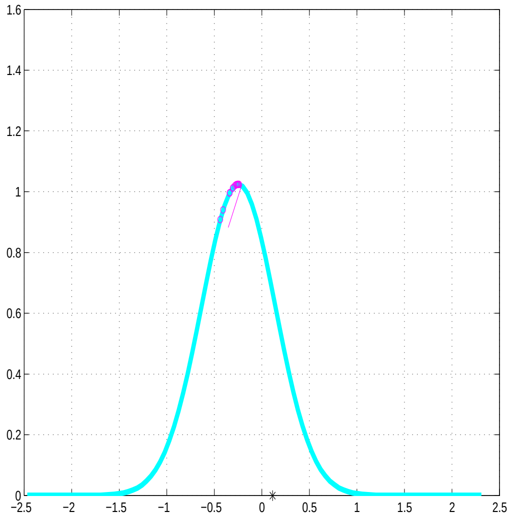
w. n g D sp t t. s r, u t on so m m xtur or asts m a st b ss s. arp t. an m ato oF n t. s ns o ntrop A stra F t orwar app at on o t. Ku ba L b r an J ns n s n qua t s a s to t. r at ons

[illegible]

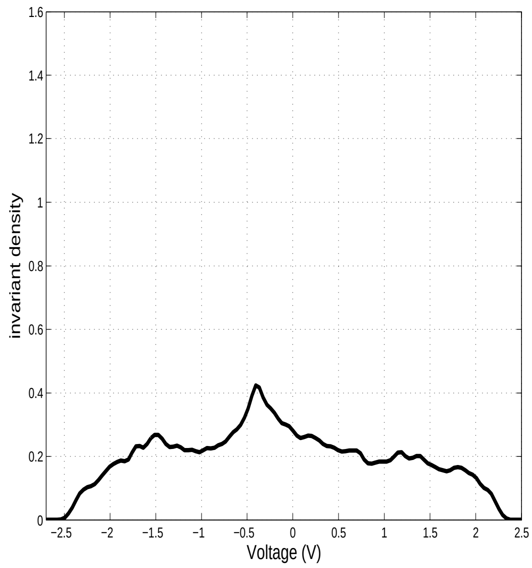
w. r $-\int$ x oF x x an $-\int$ x oF x x ar t. ntrop an ross ntrop r sp t v r or t. n ssar an su nt on t ons or [illegible] to o ar t. at [illegible] an [illegible] r sp t v n v r t. m ato oF s s. arp r t. an t. m xtur or ast t s. ou b ssu as t. or ast nst a o t. m xtur

It s not obv ous w at t. t o t. m xtur s on a brat on x pt t. at n u nF m ato oF m prov s t. KL star ro m t. a or asts v r ss t. m xtur para m t r t. at m n m s s t. oFar t. m s or s t. quat on

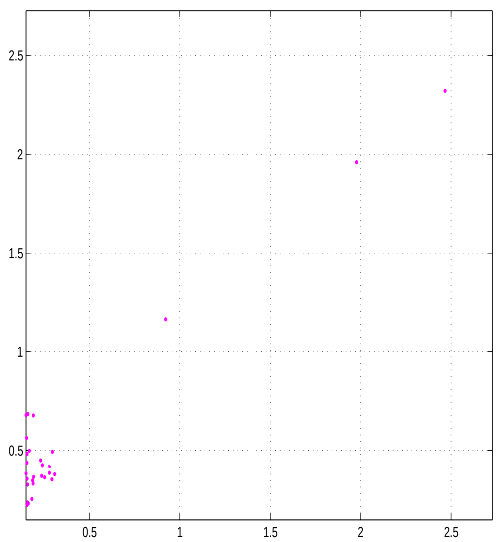
1

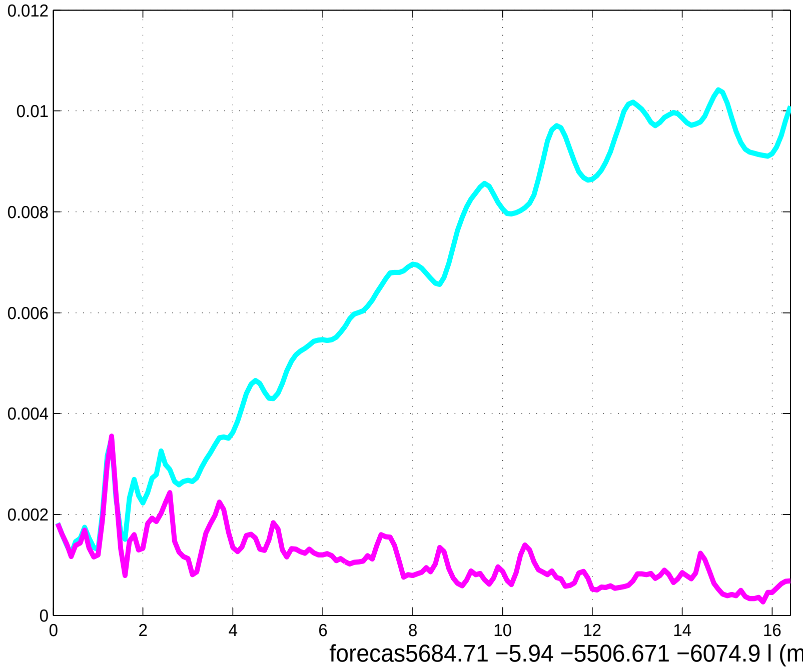


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5

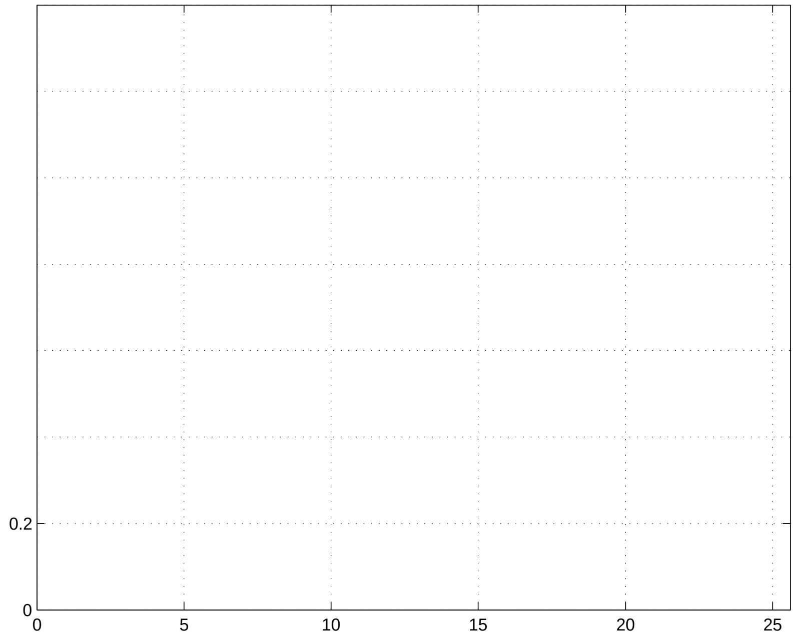


F Fur *Graph of the climatology of the circuit estimated from data. Its entropy is 2.15, which is greater than the entropies shown in figure 1.*

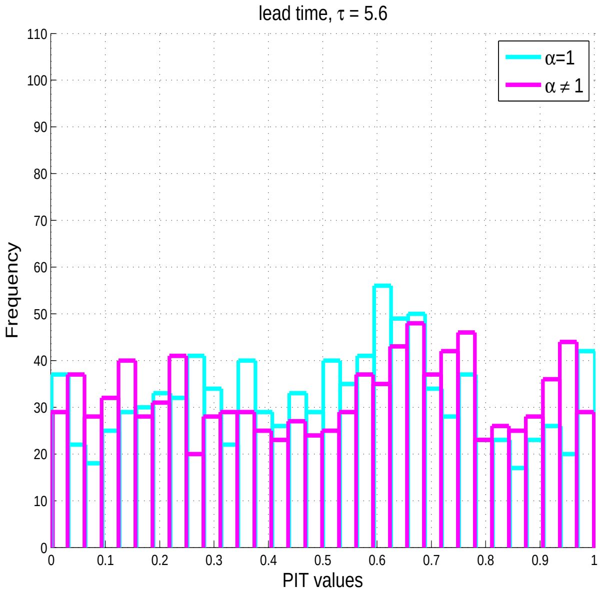




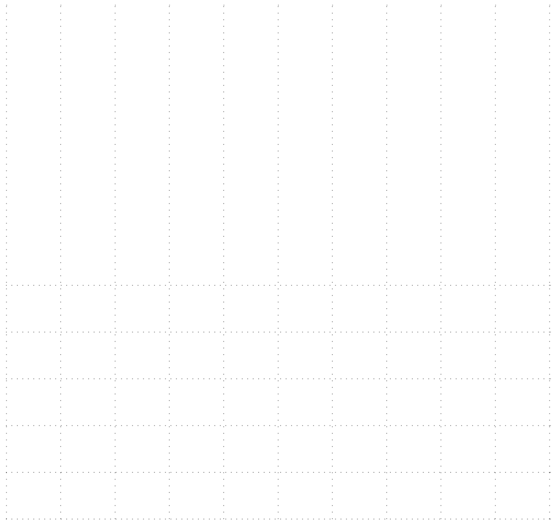
0.012
0.01
0.008
0.006
0.004
0.002
0
0
2
4
6
8
10
12
14
16
forecas5684.71 −5.94 −5506.671 −6074.9 l (m

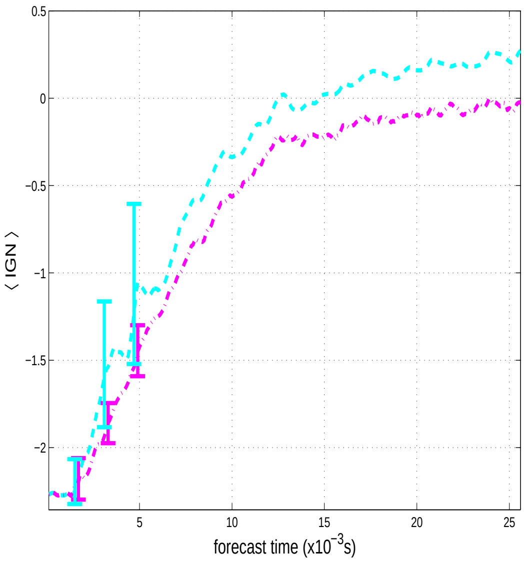


0.2
0
0
5
10
15
20
25

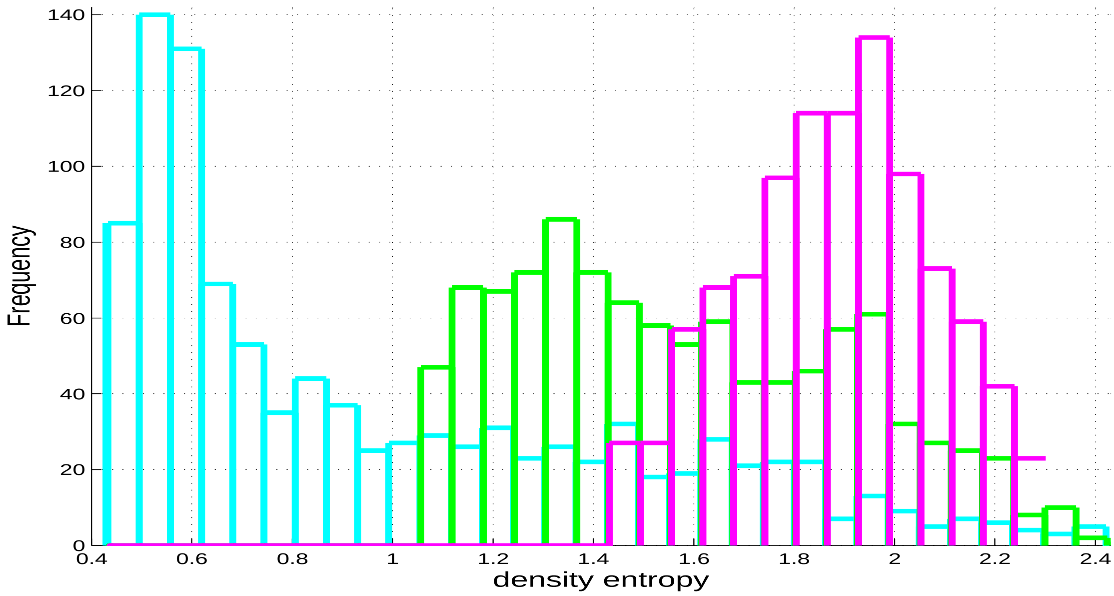


lead time, τ = 5.6
Frequency
PIT values
α=1
α ≠ 1





0.5
0
-0.5
-1
-1.5
-2
5
10
15
20
25
⟨ IGN ⟩
forecast time (x10^{-3}s)



Frequency
density entropy
0
20
40
60
80
100
120
140
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4

$\mathbf{t}\ x \quad \mathbf{t}^{-1} \quad {}_1\{\ {}_1\ x\ \}$ It t n o ows t at

$$\sum_{\mathbf{t}=1}^{\mathbf{T}} \mathbf{t} \left[\ \mathbf{t} \left\{\ \mathbf{s}^{-1}\ x_{\mathbf{s}}\ \right\}\right]$$

t r m a b at ast anot r p □ an p su t at