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Flow-Dependent Balance Conditions for Incremental Data Assimilation: Elliptic Operators

by

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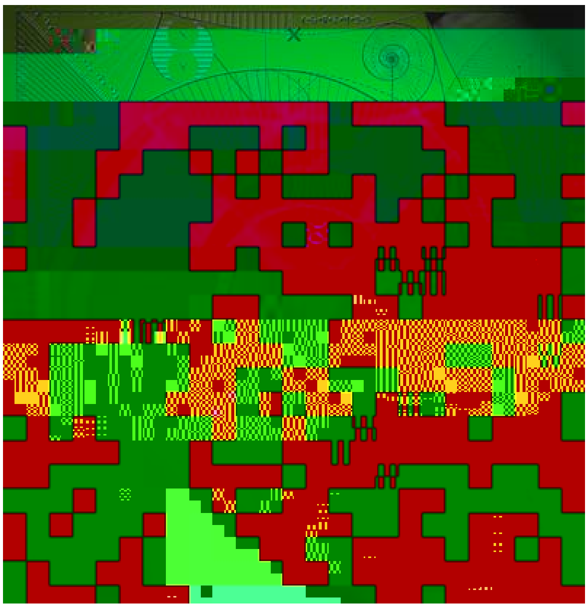

b (L *op. cit*

[illegible] *et al.*, 1[illegible]2) [illegible] 2 [illegible] (1[illegible]).

2 B[illegible] B[illegible]

[illegible] 2.1, [illegible] 2.2, [illegible] 02. [illegible] 2.[illegible]

2.1 A[illegible] B[illegible]

[illegible] *et al.* (2000). [illegible] $(u; v; p; \;) \mapsto (\;;\;;{}^{A}p)$, [illegible]

$$\equiv \cdot r \times [illegible] r^2 \; ; \quad (1[illegible])$$

$$\equiv r \cdot [illegible] r^2 \; ; \quad (1b)$$

[illegible] r [illegible].

3.2

$$u_g = -\frac{g}{af}\frac{\partial h}{\partial};\qquad v_g = \frac{g}{af}\frac{\partial h}{\partial}: \qquad (1\ )$$

fi ^{b},

$$^b \equiv \;\; - \frac{}{f}(\; \cdot r)(\;\times\;);$$

fi

$$u^b \equiv u_g - \frac{}{f}\left(\frac{u_g}{a}\frac{\partial v_g}{\partial} + \frac{v_g}{a}\frac{\partial v_g}{\partial} + \frac{}{a}u_g^2\right); \qquad (1\ )$$

$$v^b \equiv v_g + \frac{}{f}\left(\frac{u_g}{a}\frac{\partial u_g}{\partial} + \frac{v_g}{a}\frac{\partial u_g}{\partial} - \frac{}{a}u_g v_g\right); \qquad (1\ )$$

, (12).
(1) (1)
(1),

$$u^b = -\frac{g}{af}\frac{\partial h}{\partial} - \frac{g^2}{a^3 f^3 \;\; ^2}\left(\frac{\partial h}{\partial}\frac{\partial^2 h}{\partial\;\partial} - \frac{\partial h}{\partial}\frac{\partial^2 h}{\partial\;^2}\right.$$

$$+ \;\; \left(\left(\frac{\partial h}{\partial}\right)^2 + \;\; ^2 \left(\frac{\partial h}{\partial}\right)^2\right)$$

$$\left. - \;\; \frac{}{f}\left(\frac{\partial h}{\partial}\right)^2\right); \qquad (1\ )$$

$$v^b = \frac{g}{af}\frac{\partial h}{\partial} + \frac{g^2}{a^3 f^3}\left(\frac{\partial h}{\partial}\frac{\partial^2 h}{\partial\;\partial}\right.$$

$$\left. - \frac{\partial h}{\partial}\frac{\partial^2 h}{\partial\;^2} + \;\; \frac{\partial h}{\partial}\frac{\partial h}{\partial} + \frac{}{f}\frac{\partial h}{\partial}\frac{\partial h}{\partial}\right); \qquad (1\ )$$

$$\equiv \frac{df}{d}: \qquad (1\ )$$

fi

h

$$
\begin{aligned}
b \quad & \frac{g}{f} r^{2} h + \frac{g}{f^{2}a^{2}} \frac{@h}{@} + \frac{2g^{2}}{f^{3}a^{4} \quad {}^{2}} \left(\left(\frac{@^{2}h}{@ \ @} \right)^{2} \right. \\
- \quad & \frac{@^{2}h}{@ \ {}^{2}} \frac{@^{2}h}{@ \ {}^{2}} + 2 \qquad \frac{@h}{@} \frac{@^{2}h}{@ \ @} + \qquad {}^{2} \left(\frac{@h}{@} \right)^{2} \\
+ \quad & \qquad \frac{@h}{@} \frac{@^{2}h}{@ \ {}^{2}} \\
+ \quad & \frac{1}{2} \left(\left(\frac{@h}{@} \right)^{2} + \quad {}^{2} \left(\frac{@h}{@} \right)^{2} \right) \\
+ \quad & \frac{}{f} \left(2 \frac{@h}{@} \frac{@^{2}h}{@ \ {}^{2}} - 2 \frac{@h}{@} \frac{@^{2}h}{@ \ @} - 2 \qquad \left(\frac{@h}{@} \right)^{2} \right. \\
+ \quad & 2 \frac{}{f} \left(\frac{@h}{@} \right)^{2} - \frac{}{2} \qquad \left. \left(\frac{@h}{@} \right)^{2} \right) \\
- \quad & \left. \frac{1}{2f} \frac{@}{@} \left(\frac{@h}{@} \right)^{2} \right) : \qquad (1\ \ )
\end{aligned}
$$

$(1\ \ )$ h

$$\frac{@^2h}{@\ ^2}; \qquad \frac{@^2h}{@\ ^2}; \qquad \frac{@^2h}{@\ @}: \tag{22}$$

ffi A, B, C, D E (21) w (; ; h; p; q), p @h=@ ; q @h=@ .

fi (1) (20)

fi 1 > 0,

(20) 0 .

, (20), ffi

(1), fi A.

A, (20)

$$
\frac{(1+2\ \ )f}{2} + \frac{\ ^2}{fa^2}\left(v_g^2+u_g^2\right) - \frac{\ }{fa}u_g + \frac{\ ^2}{f^2a^2}u_g^2
+ \left(\frac{2\ \ ^2}{f^3a^2} - \frac{@}{@}\frac{2\ \ ^2}{f^2a^2}\right)v_g^2 > \ hq^b \ \ (f + \ ^b):
$$

f

v .

v . v h

, .

fi fi , v

h , $@h/@$ 0. w

h . / 10 1 (. (2)-10)-2 ( )

, (.)-2 ( )- 0 (

(.)- ( )- -

v w (1) (20), w

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4 .

(.) fi

fi , ^{b}

(1) 20).

.

h'

$$b' \equiv \frac{g}{f} r^2 h' + \frac{g}{f^2 a^2}\frac{\partial h'}{\partial} + \frac{2g^2}{a^4 f^3 \quad {}^2}\left(2\frac{\partial^2 \bar{h}}{\partial\ \partial}\frac{\partial^2 h'}{\partial\ \partial} \quad 2 \quad 2h\,\partial\right.$$

$$- \frac{\partial^2 \bar{h}}{\partial\ {}^2}\frac{\partial^2 h'}{\partial\ {}^2} - \frac{\partial^2 \bar{h}}{\partial\ {}^2}\frac{\partial^2 h'}{\partial\ {}^2} + 2 \quad \frac{\partial^2 \bar{h}}{\partial\ \partial}\frac{\partial h'}{\partial}$$

$$+ 2 \quad \frac{\partial \bar{h}}{\partial}\frac{\partial^2 h'}{\partial\ \partial} + 2 \quad {}^2 \quad \frac{\partial \bar{h}}{\partial}\frac{\partial h'}{\partial} + \quad {}^2 \quad \frac{\partial \bar{h}}{\partial}\ '$$

∂h
∂

4.2

5 A

v v . (0) (1) fi .
v ff fl
ffi ff
ffi ' 2- , ff
-P v (*et al.*, 1 2). ffi '
k -
(1).
-P v fi

$$
\begin{aligned}
h &= \frac{1}{g}\big(gh_0 + a^2 A(\ ) + a^2 B(\ ) \quad R \\
&\quad + a^2 C(\ ) \quad 2R \ \big); \\
u &= a! \quad + aK \quad ^{R-1} \big(R \quad ^2 - \quad ^2 \ \big) \\
&\quad \times \quad R \ ; \\
v &= -aKR \quad ^{R-1} \quad R \ ; \\
&= 2! \quad - K \quad ^{R} \big(R^2 + \ R + 2\big) \\
&\quad \times \quad R \ ;
\end{aligned}
$$

h_0 $A(\ ); B(\ )$ $C(\ )$ w

$$
\begin{aligned}
A(\ ) &= \frac{!}{2}(2\Omega + !) \quad ^2 + \frac{1}{\ }K^2 \quad ^{2R} \quad (R+1) \\
&\quad \times \quad ^2 + \big(2R^2 - R - 2\big) - 2R^2 \quad ^{-2} \ \big]; \\
B(\ ) &= \frac{2(\Omega + !)K}{(R+1)(R+2)} \quad ^{R} \big[\big(R^2 + 2R + 2\big) \\
&\quad - \ (R+1)^2 \quad ^2 \ \big]; \\
C(\ ) &= \frac{K^2}{\ } \quad ^{2R} \big[(R+1) \quad ^2 - (R+2)\big];
\end{aligned}
$$

$!$, K R
-P v , h_0.
R v ,
k v v
(P K , 1). h_0,
v , $!$ fl
K v .
v fl

ff , B_u. v

$$B_u \equiv \frac{\sqrt{gh}}{fL} = \frac{L_R}{L}; \qquad (\quad)$$

L L_R -
.

ff v B_u ff .

(2002).

fi , 1 , $h_0 =$ $0m$! =
$K =$

b – – ffi B^2

.

ffi (s^{-6})	1	2	
$\dfrac{1\ g^2 \left(\dfrac{@\bar{u}_g}{@}\right)^2}{\bar{h}^2}$	$2{:}\ \times 10^{-14}$	$:\ 1 \times 10^{-16}$	$:\ \times 10^{-18}$
$-\dfrac{2g^2\ \dfrac{@\bar{u}_g}{@}\bar{v}_g}{a\bar{h}^2}$	$-2{:}2\ \times 10^{-14}$	$-\ :\ 2 \times 10^{-16}$	$-\ :\ 2 \times 10^{18}$
$\dfrac{1\ g^2\ \ 2\ \ \bar{v}_g^2}{a^2\bar{h}^2}$	$:1\ \times 10^{-15}$	$1{:}\ \times 10^{-16}$	$1{:}\ 2 \times 10^{-18}$
B^2	$:\ \times 10^{-15}$	$1{:}\ 2 \times 10^{-16}$	$:\ \times 10^{-18}$

, 1 10^2 ffi AC . v , 2 10 . v 2 L ff b .

h_1 / :11 10 . 1 0 2 ( ) .1 3 :11 ( ) / 1 10 . -. 0 (1 -2

ffi AC

ffi (s^{-6})	1	2	
$\frac{g^2 f^2}{\bar{h}^2}$ 2	: $\times 10^{-11}$	2: $\times 10^{-14}$	:11 $\times 10^{-14}$
$\frac{g^2 f}{\bar{h}}$ $\frac{@\bar{u}_g}{@}$	h_{11}	0	

@813(i.56.15 6.74 Td[(@)]TJ /F8 99 T22)1.1388 0 Td4.882u4[(8960[(2674 Tf 5.75[.875TJ /F8 9.963 Tf 6.1363 T60[(-40 Td6.0

ffi ffi ff .

ffi (ms^{-3})		1	2	
gf	$h_{\lambda\lambda}$	$1{:}01 \times 10^{-3}$	$1{:}01 \times 10^{-3}$	$1{:}01 \times 10^{-3}$
fg	h	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$
$^{2}\ fg$	$h_{\theta\theta}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$
$2g\frac{\partial \bar{u}_g}{\partial}$	$h_{\theta\lambda}$	$2{:}\ 2 \times 10^{-5}$	$2{:}2 \times 10^{-5}$	10^{-5}
$2g\ \partial \bar{v}_g h$	$:0 \times 10^{-5}$	$2{:}0$	$\times 10$	$\times 10^{-5}$

(1) (2002).

(1), (1)

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