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State estimation using model order reduction for unstable systems

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State estimation using model order reduction for unstable systems

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a ca . *Da a a a* c a fi d b a b
c b b a a da a a ca d . I
c -d a a a a da a a (4D-Va)
a b d a a a a a b

$$_{x}\ \phi(x) = f(x)^T f(x); \qquad (1)$$

c $f(x)$ c d a ca d T
d b a a a a a Ga -N d,
a a a c a 4D-Va [1, 2]. W a d
a c a d d Jac b a c $f(x)$ a d
c Jac b a a ca d c d
a a a d (TLM). S fica ca
d c d a d a d ab b a .
I ab TLM a d a ca d
a ab a fi a T a c
b ab a a ab d b d .
A c d a ac d c c b
(1) a a TLM b a a d d a a a
. W ad a a a ac ca c

I ac c , a a a b (1) f a d fi d (2) ca b d b a Ga -N d. T a a a a ac a (k) a a a c

$$\phi(\delta x_0^{(k)}) = \|J_f \delta x_0^{(k)} + f(x_0^{(k)})\|_2^2; \tag{3}$$

J_f d Jac b a f. T a d fi d a $x_0^{(k+1)} = x_0^{(k)} + \delta x_0^{(k)}$. I (2) a Jac b a J_f b

$$J_f = \left[\left(B_0^{\frac{1}{2}}\right)^T \quad \left(R_0^{\frac{1}{2}} H_0\right)^T \quad \left(R_1^{\frac{1}{2}} H_1 M_{1;0}\right)^T \quad ::: \quad \left(R_N^{\frac{1}{2}} H_N M_{N;0}\right)^T \right]^T; \tag{4}$$

a d b a a d d a c $H_i := \frac{@h_i}{@x_i}(x^{(}$ $x^{(}$

F a a a d (k) d . W c d a - a a a a

$$\mathcal{S} : \begin{cases} \delta x_{i+1} &= M \delta x_i; \\ d_i &= H \delta x_i; \end{cases} \tag{6}$$

a (5), c a a c M a d H a a - a - a a $M_{i+1;i}$ a d H_i d $[t_0; t_N]$. T a a $\delta x_0 = B_0^{\frac{1}{2}} !$ a a d b d a d a - ab a a d c a a c a $B_0 \in \mathbb{R}^{n \quad n}$, $! \sim \mathcal{N}(0; I)$.

T a c T (6) a c a a d c d c b b a c d a . I d fi d a

$$T : \mathbb{C} \rightarrow \mathbb{R}^{p \quad m}; \tag{7}$$

$$z \mapsto T(z) := H(zI - M)^{\quad 1} B_0^{\frac{1}{2}}: \tag{8}$$

T (6) a ab , . . a a M a d a a d c c . T b ab fi d d a a (6) d a ab d d c c ab . T d α-b d d ba c d ca a c b d d [6] α-*b ded* , . . . a a a M a c c a d a ad α. F a ab $\mathcal{S}$ b fi d c a α. T α-b d d ba c d ca c c a d a a c U a d V, c , c a c d ,

$$\mathcal{S} : \begin{cases} \delta \hat{x}_{i+1} &= U^T M V \delta \hat{x}_i; \\ \hat{d}_i &= H V \delta \hat{x}_i; \end{cases} \tag{9}$$

d a c $\delta \hat{x}_i = U^T \delta x_i \in \mathbb{R}^r$, $r \ll n$, a a d $\mathcal{S}$ M c , a ba b d c $h_{1;}$ - [6]:

$$\|T - \hat{T}\|_{h_{\infty,\alpha}} \leq 2(\sigma^{(}$$

$F : \mathbb{C} \to \mathbb{R}^{m \quad p}$ a c c c c a d ad α a d, , α-b d d . T ca $\sigma^{(\ )}_{r+1}; : : : ; \sigma^{(\ )}_{n}$ a Ha a α- ca d $\mathcal{S}$, b a c

$$T \quad = \frac{1}{\alpha} H (zI - \frac{1}{\alpha} M) \quad {}^{1} B_0^{\frac{1}{2}} ;$$

d Ga -N c d . A a d c d a d aEad d $x-z$ a d ba c c ab , c d a c a a d d.

4.1. E e e a de g

T d a a 2D Ead d [7] a d c b d. T ba c a b a a d a , z, a d a b d a b da , $z = \pm\frac{1}{2}$.

F [8] a d a a c a c (QGPV) . T a a b ba b a c , $b = b(x; z; t)$, b da , $z = \pm\frac{1}{2}$, a $t = 0$. T d ca c d ba c a c- , = $(x; z; t)$, c a fi :

$$\frac{@^2}{@x^2} + \frac{@^2}{@z^2} = 0; \qquad z \in [-\frac{1}{2}; \frac{1}{2}]; \; x \in [0; X]; \tag{13}$$

b da c d

$$\frac{@}{@z} = b; \qquad z = \pm\frac{1}{2}; \; x \in [0; X]: \tag{14}$$

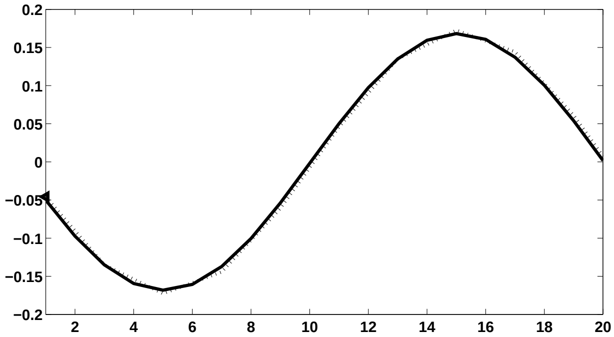
P ba ba c a a ad c d a b ba c a fl a d c b d b -d aQG d a c a :

$$\left(\frac{@}{@t} + z\frac{@}{@x}\right) b = \frac{@}{@x}; \qquad z = \pm\frac{1}{2}; \; x \in [0; X]: \tag{15}$$

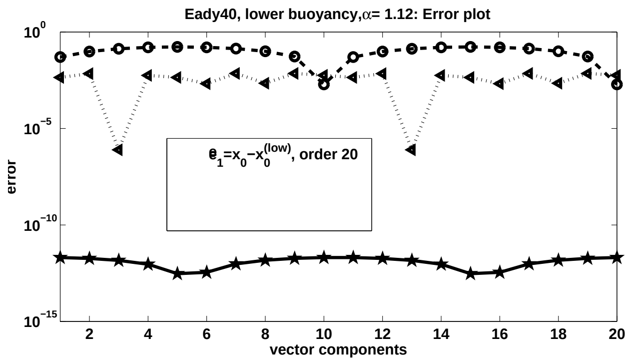
T a a b da c d x-d c a a b d c c a a a , t, a d , z, $b(0; z; t) = b(X; z; t)$ a d $(0; z; t) = (X; z; t)$. A [8] d a x, z, a d t.

I a d , Ead d d c d 11 ca a c . T a 20 d a 40 d d a c b. T ad c a a d c d a a ad c c . W [8] d a . T b a a H c c a b a a a b a c .

d c d 20, c c a a
b ab .



(a) Solution on lower boundary



(b) Error on lower boundary

Figure 1: Comparison of low resolution (dotted line with triangles), standard balanced truncation (dashtione

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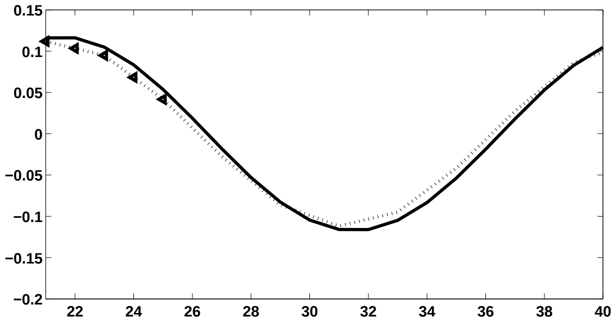

(a) Solution on upper boundary

(b) Error on upper boundary

Figure 2: Comparison of low resolution (dotted line with triangles), standard balanced truncation (dashed line with circles) and α-bounded balanced truncation (solid line with stars) approximations to the buoyancy on the upper boundary

T a c a α-b d d a - a c . T b fi ca b d a b a c d c d d a . W α-b d d d b a c fica a a ca d a d a da d ba c d ca d F 3(a) a a a d (c) F 3(b), 4(a) a d 4(b) a d ff d a a . T α-b d d a a d a c a a a d , d a a d c c (F 3(b),

c c). I c a, a da d ba c d ca d (F 4(a),
c c) ca ab a c a d

b a a a a a
d . I 4D-Va d ac d b
a a a a b c a d b a d a
a d c b a a . I
a a a ca , c b d a
a a Ga -N c d . Eac a d
c a a a a b b c a d a ,
a a d (TLM). T TLM d d c d a a
a d a b ab a fi d . T a c a
d a d a a d ab.
U a TLM a a d b a d a a a
.

I a a d a d d c d α-
b d d ba c d ca a b d b a b a a-
ab TLM Ga -N c d . T d
d c c c a d a a TLM
ca a . I ca b a d d d
b ab d . T c a
ba b d ca b d.

T d d c a a , , a d
d d d a a b a a . H ,
b a d ac ca b
K b ac c fi d c .

W a c a d α-b d d ba c d ca d
a da d ba c d ca a ac ab a d
a a ca a 2-d a
Ead d T Ead d a d ba c c ab ,
c d a c a a d d .
I ca d a c a α-
b d d a a d. I ca d a b a
d . T d a a b a c
a d b da a d d ab d . I
d, a d b a a d
a d a a a a c .

T a d a b NERC Na a C Ea
Ob a .

[1] C P, T ´ a JN, H A. A a a a - a 4D-Va , a c a a ac . Q J R M S c 1994;120:1367 87.

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