University of Reading

School of Mathematics, Meteorology & Physics

EFFICIENT EVALUATION OF HIGHLY OSCILLATORY INTEGRALS

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This dissertation is submitted to the Department of Mathematics in partial fulfilment
of the requirements for the degree of Master of Science

Declaration

I confirm that this is my own work, and the use of all material from other sources has been properly and fully acknowledged.

Acknowledgements

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1.) Introduction

$$\frac{}{\sqrt{}}\int_{\infty}^{\infty}$$

Definition 1.1

$$\int$$

Definition 1.2

$$\int$$

≠

Integral 1: Fourier Transforms

2.) Quadrature Methods

Midpoint Rule and the Composite Midpoint Rule

Definition 2.1 –

$$\int \qquad \frac{\quad}{\quad}$$

$\sum \quad \int$

$\sum \quad \int$

$\int \qquad \int \qquad \frac{\xi}{}$

$\xi\epsilon$

Definition 2.3

$\int \qquad \sum$

Lemma 2.1

≤

Example 2.1

∫

≤ | | ξ

Table 2.1

Table 2.2

Trapezium rule and the Composite Trapezium Rule

Definition 2.4-

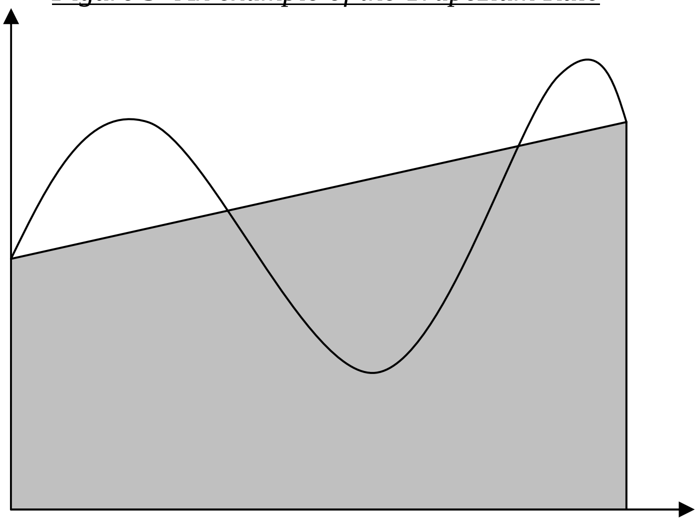
$$\int \quad - \quad -$$

Definition 2.5

$$\int \quad \approx \int$$

Proof

Figure 3- An example of the Trapezium Rule



Theorem 2.4

$\leq$ ———

———

Proof

$\int$

$\int$

Theorem 2.5 -

ξ

∫

ξ ∫ ξ

ξ ξ

ξ

ξ ξ

Definition 2.6

≤ —— ξ ϵ

——

Proof

∑ ∫

∑ ∫

Example 2.2

Table 2.3

Table 2.4

The Simpson’s Rule and the Composite Simpson's Rule

Theorem 2.7

Proof

$\int$

$\int$

Definition 2.8

$$\int \quad - \quad \sum \quad \sum$$

Proof

Table 2.5

Table 2.6

Gaussian Quadrature

Table 2.7

3) Asymptotic Methods

Definition 3.1

$$\int \qquad \sum \frac{\qquad}{\qquad} \qquad \frac{\qquad}{\qquad} \int$$

Example 3.1

Table 3.1

4) Filon Type Methods

$\sum$ —— $\sum$ ———— ————

Theorem 4.1

$\leq$

Proof

$\sum$ $\int$

$\sum$ $\int$

Table 4.1

Definition 4.2

$$\int \qquad \sum$$

$$\sum$$

Proof

$$\int \qquad \sum \int \qquad \sum \int$$

$$\int \qquad \sum \int \qquad \sum \int$$

$$\int \qquad \sum$$

$$\sum$$

Proof

$\sum \int$

$\sum \int$

______ ______ ______ ______ ______ ______ ______

$\sum$ __________

Table 4.3

Table 4.4

Filon - Gauss Method

Definition 4.3

$$\sum$$

$$\prod_{\neq} \frac{}{}$$

$$\int \qquad \int \sum$$

$$\sum \int$$

$$\int$$

$$\int \qquad \frac{\qquad}{\qquad} \quad \frac{}{} \int$$

Theorem 4.3

$$\int$$

Theorem 5.1

Proof

$$\int \qquad \int \qquad \int$$

ε

$$\int \qquad \sum \qquad \int$$

$$\int \qquad \int$$

$$\int$$

6) Filon Type Methods

Filon-Trapezoidal Method

$\neq$

Definition 6.1

$$\int \qquad \sum \quad \int \qquad \sum \quad \int$$

Case 2 $\int$ $\int$ $\neq$

$\int$

$\int$ $\int$

$\int$ $\sum$ $\int$

$\int$

$\int$ $\int$ $\int$

$\int$ $\sum$ $\int$

$\sum\sum$

Case 3 $\int$ $\int$

Conclusion

7) Bibliography and Literature Review

1) On the Numerical Quadrature of Highly-Oscillating Integral I: Fourier transforms - Arieh Iserles IMA Journal of Numerical Analysis 2004 p365-391

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