

University of Reading

Modelling of Hot Water Flooding

as an

Enhanced Oil Recovery Method

by

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August 2013

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Submitted to the Department of Mathematics, University of Reading,
in Partial fulfilment of the requirements for the Degree of Master of Science in
Mathematics of Scientific and Industrial Computation

Acknowledgment

Declaration

I confirm that this is my own work and the use of all materials from other sources has been properly and fully acknowledged.

Signed

Abstract

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1 Introduction

A hydrocarbon reservoir is an underground volume comprised of porous rock containing a mixture of water and hydrocarbon fluids in the form of oil and gas, occupying the void space of the pores in the rock. Oils can be divided into two categories, light oils and heavy oils. Light oils have a low viscosity while heavy oils have a high viscosity. The viscosity of a fluid is a measure of how easily that fluid will flow, for instance, water has a very low viscosity while honey has a high viscosity.

When oil recovery is high due to high natural reservoir pressure. The rate of natural oil production will diminish with time, but there are some oil recovery methods to improve the production rate. Oil recovery processes involve the injection of fluid or a combination of fluid and chemicals into the oil reservoir via injection wells to force as much oil as possible towards and, hence, out of the production wells. Light oils are extracted under primary and secondary recovery methods which involve allowing the fluid to flow out under the natural pressure of its surrounding. These methods cannot be applied to the extraction of heavy oils, whose viscosity is far too high for such methods to be effective; their viscosity needs to be reduced. This is achieved by various thermal stimulation techniques like hot water flooding, steam injection, in-situ combustion and so far which raise the temperature of the oil, effectively reducing its viscosity. The approach which is under consideration here is hot water injection modeling. It is necessary to model and simulate this process in order to provide information about production and the future of the reservoir to get the best recovery.

All thermal recovery processes tend to raise the temperature of the crude in a reservoir to reduce the reservoir flow resistance by reducing the viscosity of the crude [12]. It is desirable to heat the reservoir efficiently, but inevitably some of the heat in the reservoir is lost through produced fluids, and some is lost to the adjacent overburden and underburden formations. The heat loss to the adjacent formations is controlled by conduction (heat transfer) which it can be readily estimated.

In hot water flooding, as can be seen in figure 1.1 many reservoir equivalent volumes of hot water are injected into a number of wells in order to reduce the viscosity and subsequently displace the oil in place more easily towards oil production wells. Hot water injection may be preferred in shallow reservoirs containing oils in the viscosity range of 100-1000 cp [4].

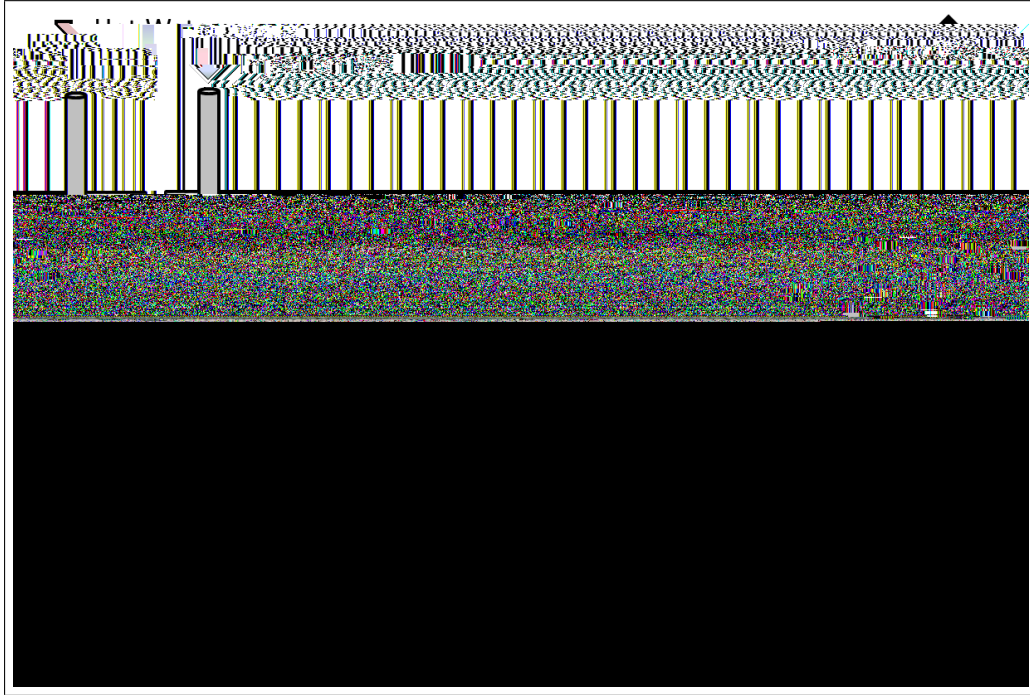


Figure 1.1: Schematic diagram of hot water injection process

The mathematical model representing the physical process of hot water injection requires rock and fluid properties in order to describe the fluid flow and heat transfer with a set of partial differential equations and algebraic equations, which are derived from physical principals. This set of equations is derived from four main principles: Conservation of mass of phases (water and oil); Darcy's Law for volumetric flow rates which describes how the fluid phases flow through the reservoir; volume balance equation, a condition which states that the fluid fills the rock pore volume; conservation of energy of phases. Since the resulting equations are too complex for more realistic models to be solved using analytic techniques, here is focused on numerical techniques.

In this dissertation, two different models are applied and analyzed for the hot water injection process. Chapter 2 contains the problem definition and characteristics of the model. Introducing some necessary concepts about rock and fluid properties, and required equations. Initial and boundary conditions and heat loss in our model are also included in this part. The first model is introduced in chapter 3, where in order to find the saturation distribution the Buckley-Leverett equation is used. The nonlinear mass balance equation is solved by a fully implicit central technique by using the results of oil and water saturations from the Buckley-Leverett equation. Subsequently, the saturation and pressure results are applied to a nonlinear energy equation discretized by a fully implicit method. Finally, the mass balance equation

2 Characteristics of the Model

In this project, we have tried to model the hot water flooding process in a reservoir which is initially saturated with oil and water. The reservoir is considered to be one-dimensional between an injection and production wells. A schematic diagram of the model is given in figure 2.1. Hot water is injected with a constant rate and temperature into the porous media which is filled with cold and heavy oil. In such a system fluid flow, heat transfer and heat losses are modeled in order to give a better understanding of the process and its effect on oil recovery.

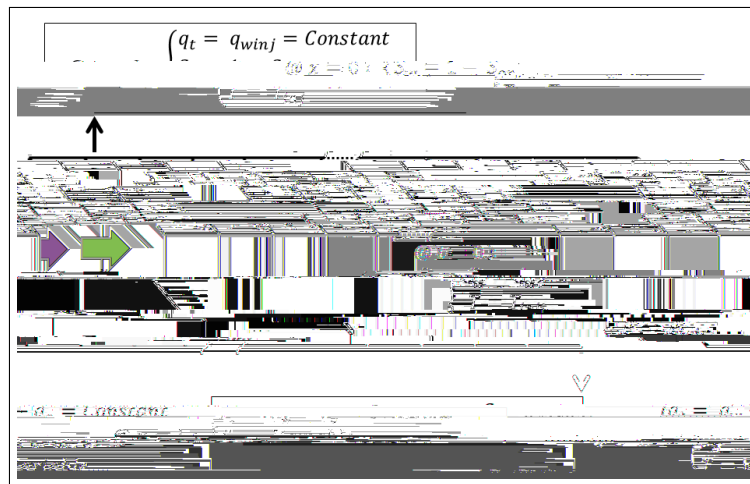


Figure 2.1: Schematic diagram of the problem

2.1 Assumptions

The following assumptions are made to model the process;

1. In all reservoir processes, every point within the reservoir is in thermodynamics equilibrium.
2. The injected fluid reaches thermal equilibrium instantaneously with the reservoir fluids and sand, meaning that all phases and rock in the same location have the same temperature.

$$V_p = \frac{\text{Pore Volume } (V_p)}{\text{Bulk Volume } (V_b)} \quad (2.4)$$

2.2.3 Saturation

The pore volume space is not always filled with a single fluid. Saturation of each fluid (phase) is defined as the ratio of its volume over the total pore volume occupied by all phases [1],

$$S_i = \frac{\text{Phase Volume } (V_i)}{\text{Pore Volume } (V_p)} \quad (2.5)$$

By definition, the saturations are all non-negative, and sum to one.

2.2.4 Permeabilities

One of the main properties of porous rock is its capability to allow fluid flow through its connected pores which is known as permeability. There are two definitions of permeability in the oil industry; absolute and relative permeabilities. Under the condition of single phase flow, this capability is named absolute permeability. But when the porous media is filled by more than one phase, due to various ways the phases can occupy the pore volume, the phases adversely affect the flow of each other in a complicated manner [1]. This effect is described using phase relative permeabilities, K_{ro} and K_{rw} . The dependence of the relative permeabilities on the rock and fluid properties is very complicated [2]; the K_{ro} and K_{rw} considered here are non-negative functions of the saturation, S definition, t ls

S_{wi} : Irreducible water saturation
 S_{wc} : Connate water saturation
 S_{orw} : Residual water saturation (water-oil system)
 S_w : Water saturation
 S_{wn} : Normalized water saturation

Figure 2.2 shows the results of water and oil relative permeabilities versus water saturation in the system by applying the Corey correlation. In describing two-phase flow mathematically, it is always the relative permeability ratio, $\frac{K_{ro}}{K_{rw}}$, versus water saturations (for oil and water system) that enters the equations.

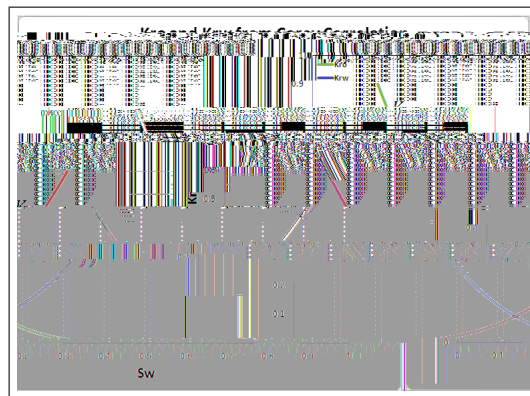


Figure 2.2: Oil and water relative permeabilities using Corey correlation

2.2.5 Hydrocarbon Viscosity

Phase viscosity represents the resistance of a phase to flow under the influence of a pressure gradient. The most obvious effect of thermal recovery on a reservoir fluids is the reduction of oil viscosity. In Figure 2.3 two points are evident. First, the rate of viscosity improvement is greatest as the initial temperature increases. Little viscosity benefit is gained after reaching a certain temperature. Second, greater viscosity reductions are experienced in the more viscous low API gravity crudes (API is a degree of measurement for oil density) than in higher API gravity crudes. Heating from 100°F to 200°F reduces the viscosity, 98% for 10°API crudes but only 73% for 30°API oils. These observations show that the greatest viscosity reduction occurs with the more viscous oils at the initial temperature increases [4].

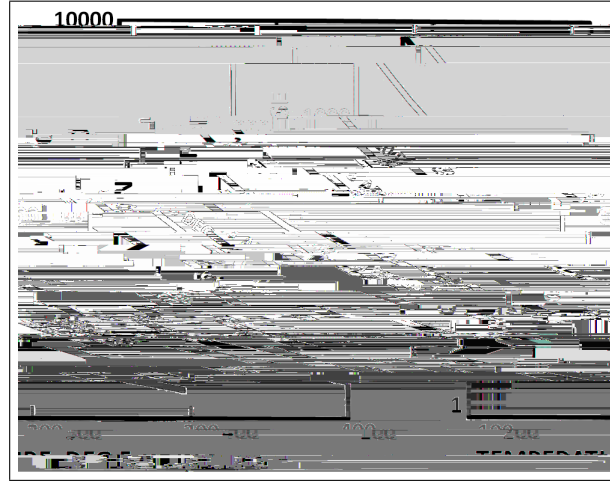


Figure 2.3: Effect of temperature on viscosity

Viscosity is a function of temperature and pressure, but water and oil viscosities are stronger functions of temperature in a thermal process rather than pressure. Since a thermal oil recovery method is modeled in this project, the effect of pressure is neglected [5].

$$\mu_o = 2.626 \cdot 10^8 (T - 459.59)^{-2.91} \quad (2.9)$$

$$\mu_w = \frac{2.185}{0.04012 (T - 459.59) + 5.154 \cdot 10^{-6} (T - 459.59)^2} \quad (2.10)$$

2.2.6 Phase Mass Density

Phase density is defined as mass per unit volume for each phase. Water and oil densities in this context are considered to be a function of temperature and pressure. In the absence of experimental data, empirical relations are used to express densities of oil and water as functions of both temperature and pressure [5]:

$$\rho_w = 63 \exp(-17.253 \cdot 10^{-5} (T - 459.59)) \exp(-4 \cdot 10^{-6} (P - 1000)) \quad (2.11)$$

$$\rho_o = 59 \exp(-7.5885 \cdot 10^{-5} (T - 459.59)) \exp(-1 \cdot 10^{-5} (P - 1000)) \quad (2.12)$$

Figure 2.4 shows the effect of temperature and pressure increase on the densities of both phases.

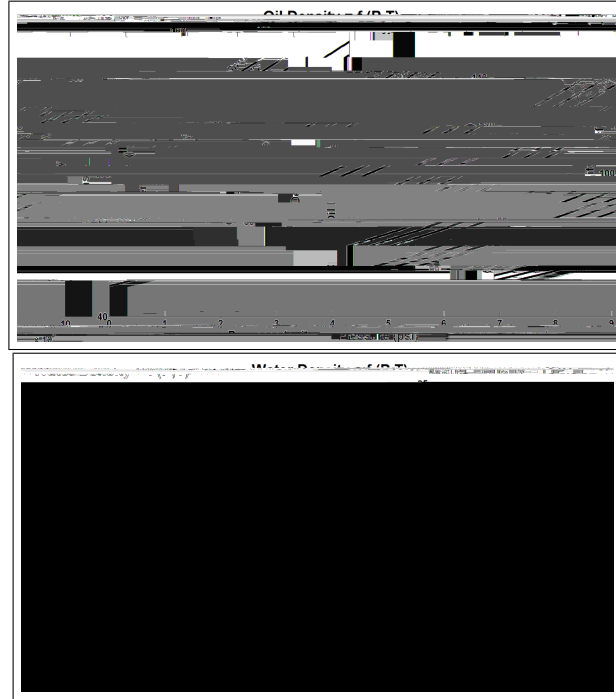


Figure 2.4: Effect of temperature and pressure on oil and water densities

2.3 Introducing Model Equations

In the two hot water models presented in this dissertation, four equations are required; the Buckley-Leverett equation, mass balance equations for water and oil, and an energy Balance equation.

2.3.1 Buckley-Leverett

The Buckley-Leverett (BL) equation is used in oil recovery in order to find the saturation distribution in 1D reservoir. In the BL mechanism oil is displaced by water from a rock in a similar as fluid is displaced from a cylinder by a leaky piston. In order to have better understanding of the Buckley-Leverett equation, it is first necessary to introduce the fractional flow equation.

2.3.1.1 Derivation of Fractional Flow for the Model

When oil is displaced by water in the system, from Darcy's equation we have

$$q_w = \frac{1}{1.27} K_{abs} K_{rw}$$

$$q_o = 1:127 K_{abs} \frac{K_{ro}}{o} A_x \frac{@P}{@x} \quad (2.14)$$

By adding the two equations

$$q_w + q_o = 1:127 K_{abs} A_x \left(\frac{K_{rw}}{w} + \frac{K_{ro}}{o} \right) \frac{@P}{@x} \quad (2.15)$$

Substituting for

$$q = q_w + q_o \quad (2.16)$$

and

$$f_w = \frac{q_w}{q} \quad (2.17)$$

and solving for the fraction of water flowing, we obtain

$$f_w = \frac{1}{1 + \frac{K_{ro}}{o} : \frac{w}{K_{rw}}} \quad (2.18)$$

since

$$K_r(S_w) = f_w(S_w) \quad (2.19)$$

Now, the Buckley-Leveret equation is derived for a 1D sample based on mass conservation and some assumptions [6], namely flow is linear and steady state, the fluid is incompressible, capillary pressure (P_c) is just a function of the saturation and pressure gradient for two phases is equal ($\frac{dP_c}{dS} = 0$), where $P_c = P_o - P_w$.

By applying mass balance of water around a control volume (see figure 2.5) of length Δx we get the following system for a time period of Δt :

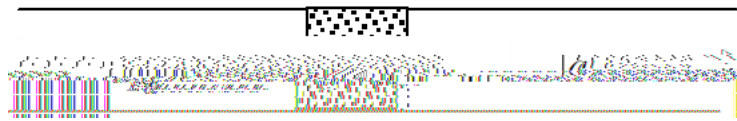


Figure 2.5: Mass Balance Element for Fractional Flow Equation

The material balance may be written:

$$\int_{x}^{x+\Delta x} (w q_w)_{t+\Delta t} - \int_{x}^{x+\Delta x} (w q_w)_t = A \Delta x \left[(w S_w)_{t+\Delta t} - (w S_w)_t \right] \quad (2.20)$$

which, when $\Delta x \rightarrow 0$ and $\Delta t \rightarrow 0$, reduces to the continuity equation:

$$\frac{@}{@x} (w q_w) = A \frac{@}{@t} (w S_w) \quad (2.21)$$

by assuming an incompressible fluid $\rho_w = \text{constant}$ and we have that $q_w = f_w q$,
Therefore

$$\frac{\partial f_w}{\partial x} = \frac{A'}{q} \frac{\partial S_w}{\partial t} \quad (2.22)$$

Since $f_w(S_w)$, equation 2.22 may be rewritten as [6]

$$\frac{\partial f_w}{\partial S_w} \frac{\partial S_w}{\partial x} = \frac{A'}{q} \frac{\partial S_w}{\partial t} \quad (2.23)$$

where fractional water flow is defined as

$$f_w(S_w) = \frac{1}{1 + \frac{K_{ro}}{K_w} \frac{1 - S_w}{S_o}} = \frac{1}{1 + \frac{(K_{ro}^{max})}{K_w^{max}} \frac{1 - S_n}{S_n} \frac{1 - S_o}{S_o}}$$

$$S_n = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \quad S_{wc}$$

2.3.2 Mass Balance Equation

The main flow equation in reservoir engineering can simply be derived by applying material balance to a control volume, as shown in figure 2.7. Mass accumulation inside a control volume is the difference between input and generated mass and output and consumed mass as below:

$$(\bar{m}_i - \bar{m}_o) - (\bar{m}_{\text{cons}} - \bar{m}_{\text{gen}}) = \frac{\partial M}{\partial t} \quad ; \quad \bar{m} = \text{Mass Flux} = \rho : q \quad (2.26)$$

where q is flow rate, ρ is density, M is mass and t is time. Time

Based on what we have in equation 2.26 for 1D flow, the input mass rate for x direction shown in figure 2.7 will be:

$$\bar{m}_{ix} = \rho : u_x : dA_x \quad (2.27)$$

dA_x , u_x and ρ are the normal cross sectional area in x direction, velocity and density respectively.

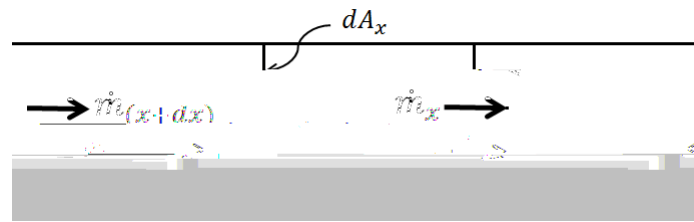


Figure 2.7: Material Balance Control Volume

Using the Euler approximation for the mass rate

$$\bar{m}_{i(x+dx)} - \bar{m}_o = \bar{m}_{i(x)} + \frac{\partial \bar{m}_{ix}}{\partial x} : dx \quad (2.28)$$

The generation and consumption terms in reservoir engineering are production and injection in wells and can be specified as:

$$\begin{aligned} \bar{m}_{\text{cons}} &= \rho : q_{\text{prod}} \quad ; \quad q_{\text{prod}} = \text{Production Rate} \\ \bar{m}_{\text{gen}} &= \rho : q_{\text{inj}} \quad ; \quad q_{\text{inj}} = \text{Injection Rate} \end{aligned} \quad (2.29)$$

By substituting equations 2.27, 2.28 and 2.29 in equation 2.26:

$$\frac{\partial}{\partial x} (\rho : u_x : dy : dz) : dx - (\rho : q_p - \rho : q_i) = \frac{\partial}{\partial t} (\rho : dx : dy : dz) \quad (2.30)$$

Using $V_b = dx : dy : dz$ and dividing both sides to V_b

Equation 2.31 is the most general type of the mass conservation law in its one dimensional form [8]. To make it more usable in reservoir engineering, Darcy law (equation 2.3) is used to substitute the velocities. Hence, the result for multi-phase flow in porous media will be

$$\frac{\partial}{\partial x} \left(K_{abs} K_r \frac{\partial P}{\partial x} \right) - \frac{\partial}{\partial t} (\phi S) = \frac{\partial}{\partial t} (\phi S)$$

$$e_{(x+dx)} - e_{ox} = -k \left(\frac{\partial T}{\partial x} + u_x H \right) + \frac{\partial}{\partial x} \left(k \left(\frac{\partial T}{\partial x} + u_x H \right) \right) dx dy dz \quad (2.36)$$

$$e_{ix} - e_{ox} = -k \frac{\partial^2 T}{\partial x^2} dx dy dz + \frac{\partial}{\partial x} (u_x H) dx dy dz = -k \frac{\partial^2 T}{\partial x^2} dx dy dz + \frac{\partial}{\partial x} (K H \frac{\partial P}{\partial x}) dx dy dz \quad (2.37)$$

For wells, the heat transfer can be divided into conduction and convection based on bottom hole temperature and fluid enthalpy, as below:

$$e_w = e_{cons} \quad e_{gen} = (q_{inj} - q_{prod}) H - (2 \pi k h) r_w \left. \frac{\partial T}{\partial r} \right|_{r=r_w} \quad (2.38)$$

For the accumulation term in equation 2.33, both the rock and fluid must be taken into account as they both have heat capacities and are able to store energy in themselves. Therefore the accumulation term will be:

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t} ((\rho_o U_o S_o + \rho_w U_w S_w)) dx dy dz + \rho_r U_r (1 - \phi) dx dy dz \quad (2.39)$$

Now all the above equations must be combined

supposed that two wells are located at two sides or boundaries of the reservoir. An injection well with constant rate (or total rate) and injection temperature (T_{inj})

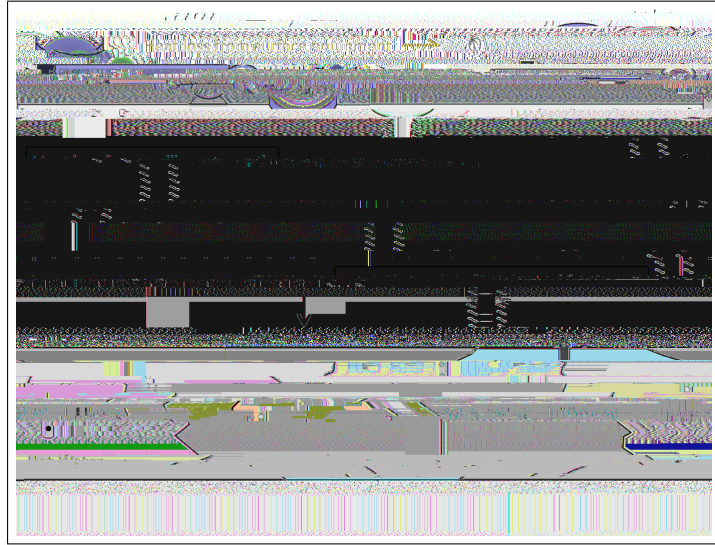


Figure 2.9: Illustration of heat losses which occur in a heat injection system

3 First Model

In this model, the Buckley-Leverett equation is used to find saturation profiles and then its results are used in the pressure equation which is solved by a fully implicit numerical technique. In order to find the temperature profile the saturation and pressure results are applied to a fully implicit energy equation. Finally, the mass balance equation (pressure equation) and the energy balance equation (temperature equation) are coupled to find the optimal result for pressure and temperature distributions, since these equations are highly nonlinear.

3.1 Buckley-Leverett Discretization

In this model, the Buckley-Leverett equation is used to find the saturation distribution. The numerical scheme used to solve this hyperbolic equation is the Lax-Wendroff scheme. The scheme is a second order finite difference method where the derivatives are approximated by differences of discrete values. An important requirement of numerical methods for such nonlinear hyperbolic equations is to be in conservative form to maintain the conservation property of the equation. To derive the numerical method in conservative form we use standard finite difference discretization of the conservative form of the partial differential saturation equation, not the quasilinear form of the equation.

For a numerical scheme to be in conservation form [9] it must have the form

$$U_j^{n+1} = U_j^n - \frac{4\Delta t}{4\Delta x} [F(U_{j+\frac{1}{2}}^{n+\frac{1}{2}}) - F(U_{j-\frac{1}{2}}^{n+\frac{1}{2}})]g; \quad (3.1)$$

where U_j^n is an approximation to the cell average of the analytic function, $F(U_{j+\frac{1}{2}}^{n+\frac{1}{2}})$ is the numerical flux function, $4\Delta t = t^{n+1} - t^n$, and $4\Delta x = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}$.

By using oil field units (refer to nomenclature), the Buckley-Leverett equation will be,

$$\frac{A'}{5.615q} \frac{\partial S}{\partial t} + \frac{\partial f}{\partial x} = 0 \quad (3.2)$$

By choosing,

$$\begin{aligned} X &= \frac{x}{L} \Rightarrow dX = \frac{1}{L} dx \Rightarrow \frac{\partial}{\partial x} = \frac{1}{L} \frac{\partial}{\partial X} \\ t_D &= \frac{5.615 q_t}{A \cdot L} t \Rightarrow dt_D = \frac{5.615}{A \cdot L} dt \\ \Rightarrow \frac{\partial}{\partial t} &= \frac{5.615 q_t}{A \cdot L} \frac{\partial}{\partial t_D} \end{aligned} \quad (3.3)$$

the BL equation converts into the dimensionless equation,

$$\frac{\partial S}{\partial t_D} + \frac{\partial f}{\partial X} = 0 \quad (3.4)$$

In order to drive the Lax-Wendro scheme applied to above equation to be in conservative form, for all intermediate blocks ($i = 2; \dots; N_x - 1$), we start with;

$$\frac{\partial S}{\partial t_D} + \frac{\partial f}{\partial X} = 0 \Rightarrow \frac{\partial S}{\partial t_D} = - \frac{\partial f}{\partial X} \quad (3.5)$$

and using Taylor-series expansion about t_D

$$S(X_i; t_D + \Delta t_D) = S(X_i; t_D) + \Delta t_D \frac{\partial S(X_i; t_D)}{\partial t_D} + \frac{\Delta t_D^2}{2} \frac{\partial^2 S(X_i; t_D)}{\partial t_D^2} + O(\Delta t_D)^3 \quad (3.6)$$

$$\begin{aligned} \frac{\partial S}{\partial t_D} &= \frac{\partial f}{\partial X} \\ \Rightarrow \frac{\partial S}{\partial t_D} &= (f_S)_{t_D} = (f_{t_D})_X = \frac{\partial f}{\partial S} \frac{\partial S}{\partial t_D} \bigg|_X = \frac{\partial f}{\partial S} (f_X) \bigg|_X = \frac{\partial f}{\partial S} \frac{\partial f}{\partial X} \bigg|_X \end{aligned} \quad (3.7)$$

By substituting central differences for space derivatives

$$\begin{aligned} S_i^{n+1} &= S_i^n + \Delta t_D \frac{f_{i+1/2} - f_{i-1/2}}{h} + \frac{\Delta t_D^2}{2} \frac{(\frac{\partial f}{\partial S} \frac{\partial f}{\partial X})_{i+1/2} - (\frac{\partial f}{\partial S} \frac{\partial f}{\partial X})_{i-1/2}}{h} \\ S_i^{n+1} &= S_i^n - \frac{\Delta t_D}{h} (f_{i+1/2} - f_{i-1/2}) + \frac{\Delta t_D^2}{2} \frac{f_{i+1/2} - f_{i-1/2}}{h} \end{aligned} \quad (3.8)$$

: Conservative form

) By comparison:

$$\frac{\Delta t_D}{h} = \frac{1}{2} (f_{i+1/2} + f_{i-1/2}) \quad (3.9)$$

where

$$\begin{aligned} \frac{\Delta t_D}{h} &= \frac{1}{2} (f_{i+1/2} + f_{i-1/2}) \\ \Rightarrow \frac{\Delta t_D}{h} &= \frac{1}{2} (f_{i+1/2} + f_{i-1/2}) \end{aligned} \quad (3.10)$$

Similarly $h_{i-\frac{1}{2}} = \frac{1}{2}(f_i + f_{i-1}) - \frac{1}{2}(f_i - f_{i-1})$

$$= \begin{cases} -\frac{t_D}{X} \frac{f_i - f_{i-1}}{S_i - S_{i-1}} & \text{if } S_i \neq S_{i-1} \\ -\frac{t_D}{X} \frac{\partial f}{\partial S} \bigg|_{S_{i-1}} & \text{if } S_i = S_{i-1} \end{cases} \quad (3.11)$$

3.1.1 Effect of boundary conditions

For the first cell ($i = 1$), it is supposed that there is a known value of S at the boundary ($S = 1 = S_{or}$). At this point the Lax-Wendro scheme is derived based on the unequal spacing (see figure 3.1)

Figure 3.1: Unequal spacing for the left boundary cell after introducing an imaginary node

$$\begin{aligned} & f(X_1 + X) = f(X_1) + X \frac{\partial f}{\partial X} \bigg|_{X_1} \\ & f(X_1 - X_b) = f(X_1) - X_b \frac{\partial f}{\partial X} \bigg|_{X_1} \end{aligned} \quad \Rightarrow \quad \frac{\partial f}{\partial X} \bigg|_{X_1} = \frac{f_2 - f_b}{X + X_b} \quad (3.12)$$

$$S_1^{n+1} = S_1^n - t_D \frac{f_2 - f_b}{X + X_b} + \frac{t_D^2}{2} \frac{\partial}{\partial S} \left(\frac{\partial f}{\partial X} \bigg|_{1 + \frac{X}{2}} \right) S$$

where

$$\begin{aligned}
 1 + \frac{x}{2} &= \frac{t_D}{X} \cdot \frac{f_2}{S_2} \cdot \frac{f_1}{S_1} S_2 \neq S_1 \\
 &= \frac{t_D}{X} \cdot \frac{\partial f}{\partial S_1} S_2 = S_1 \\
 1 - \frac{x_b}{2} &= \frac{t_D}{X_b} \cdot \frac{f_1}{S_1} \cdot \frac{f_b}{S_b} S_b \neq S_1 \\
 &= \frac{t_D}{X_b} \cdot \frac{\partial f}{\partial S_b} S_b = S_1
 \end{aligned} \tag{3.16}$$

All the fractional water values and its derivatives in above formulas are calculated from equations 2.24 and 2.25

For last cell ($i = N_x$), the equation 3.4 is discretized based on forward in time and backward in space as follows,

$$\frac{S_i^{n+1} - S_i^n}{t_D} + \frac{f(S_i^n) - f(S_{i-1}^n)}{X} = 0 \tag{3.17}$$

$$) \quad S_i^{n+1} = S_i^n - \frac{t_D}{X}$$

the stencil of the scheme, whilst the line AC violates the CFL condition, by lying outside the domain of dependence.

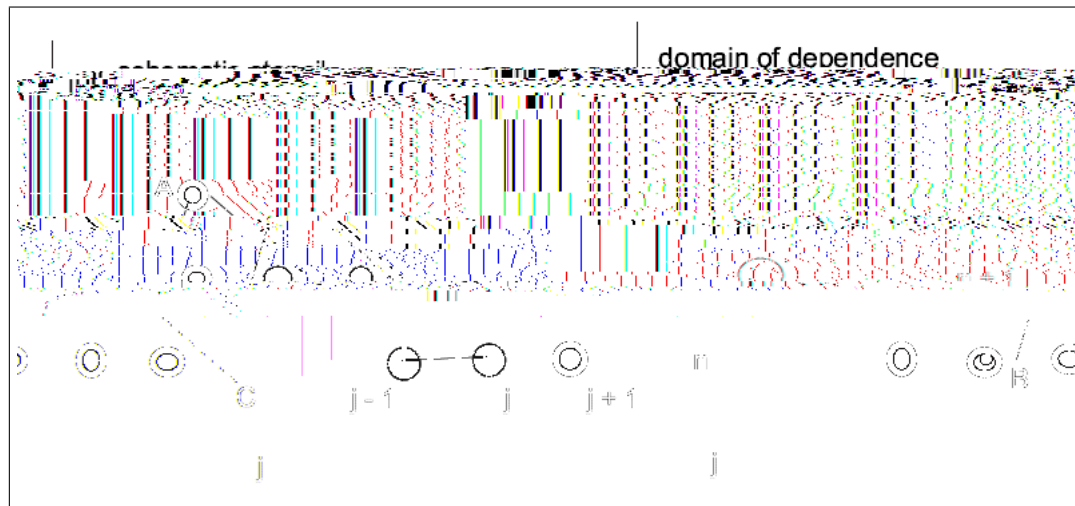


Figure 3.3 shows the grid indexing scheme used for the method. The above equation is discretized as,

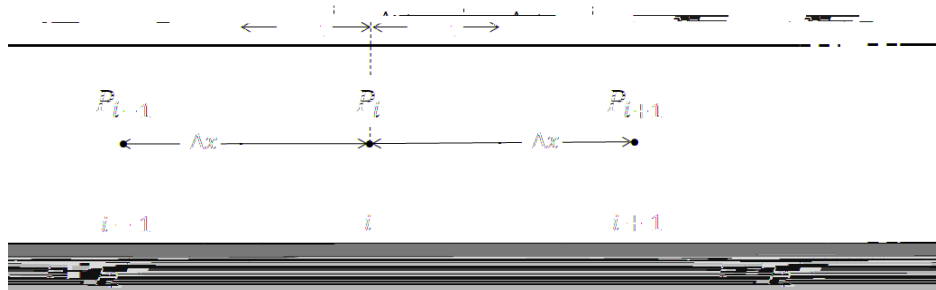


Figure 3.3: Grid indexing scheme in material balance equation

$$\frac{\left(\frac{\partial K_o}{\partial p} \right)_{i+\frac{1}{2}}^{n+1} - \left(\frac{\partial K_o}{\partial p} \right)_{i-\frac{1}{2}}^{n+1}}{\Delta x} + \phi_i \frac{\partial p}{\partial t} = 0$$

$$D_{oi+1}^{n+1} \cdot \frac{p_{i+1}^{n+1} - p_i^{n+1}}{6:328 \cdot t} + \frac{D_{oi}^{n+1} \cdot p_i^{n+1} - D_{oi+1}^{n+1} \cdot p_i^{n+1}}{6:328 \cdot t} + \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{opi} \cdot p_i^{n+1} - p_i^n + 1 \cdot S_i^{n+1} \cdot \frac{0}{oTi} \cdot T_i^{n+1} - T_i^n \right) - \frac{oi}{6:328 \cdot t} \left(S_i^{n+1} - S_i^n \right) \quad (3.27)$$

So, general oil pressure equation for each middle cell ($i=2, \dots, N_x - 1$) is defined as a function of three variable p_{i+1} , p_i and p_{i-1} ,

$$D_{oi}^{n+1} \cdot \frac{p_i^{n+1} - p_{i-1}^{n+1}}{6:328 \cdot t} + \frac{D_{oi}^{n+1} + D_{oi+1}^{n+1}}{6:328 \cdot t} \cdot \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{opi} \cdot p_i^{n+1} + D_{oi+1}^{n+1} \cdot p_{i+1}^{n+1} \right) = \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{opi} \cdot p_i^n + \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{oTi} \cdot T_i^{n+1} - T_i^n \right) - \frac{oi}{6:328 \cdot t} \left(S_i^{n+1} - S_i^n \right) \right) \quad (3.28)$$

Treating the equation for boundary cells is slightly different. For the first cell ($i = 1$), the equation 3.23 is discretized as

$$\frac{\frac{0}{K_o} \cdot \frac{@P}{@x} \cdot \frac{n+1}{i+\frac{1}{2}} - \frac{0}{K_o} \cdot \frac{@P}{@x} \cdot \frac{n+1}{bl}}{x} + \frac{oi \cdot q_o}{1:127 V_{bi}} = \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{opi} \cdot p_i^{n+1} - p_i^n + 1 \cdot S_i^{n+1} \cdot \frac{0}{oTi} \cdot T_i^{n+1} - T_i^n \right) - \frac{oi}{6:328 \cdot t} \left(S_i^{n+1} - S_i^n \right) \quad (3.29)$$

Using equation 2.43 and boundary conditions help to find water properties and boundary pressure gradient in above equation,

$$\begin{aligned} S_b &= 1 - S_{or} ; T_b = T_{inj} ; p_b = p_{bl} \\ K_{wbl} &= K_{abs} \cdot K_{rw} (1 - S_{or}) = K_{abs} \cdot K_{rw}^{max} \\ w_b &= w(P_{bl}; T_{inj}) \\ \frac{P_i - P_{bl}}{x/2} &= \frac{@P}{@x}_{bl} = \frac{q_{t, wbl}}{1:127 K_{wbl} A_n} = G P I \end{aligned} \quad (3.30)$$

Note that the injection well is located on the boundary, so the effect of generation terms is considered in the boundary pressure gradient. Hence, the discretized oil pressure equation for the first cell ($i = 1$) is

$$D_{oi+1}^{n+1} + \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{opi} \cdot p_i^{n+1} + D_{oi+1}^{n+1} \cdot p_{i+1}^{n+1} \right) = D_{oi}^{n+1} \cdot x \cdot G P I + \frac{1}{6:328 \cdot t} \left(S_i^{n+1} \cdot \frac{0}{opi} \cdot p_i^n + \frac{(1 - S_i) \cdot \frac{0}{oTi} \cdot T_i^{n+1} - T_i^n}{6:328 \cdot t} - \frac{oi}{6:328 \cdot t} \left(S_i^{n+1} - S_i^n \right) \right) \quad (3.31)$$

Condition at the right boundary cell is different. Using equation 2.44 and defining a new parameter, K_{rb} , the pressure gradient for this boundary is calculated as

$$K_{rb} = \frac{K_{rob}}{\Delta x_{ob}} + \frac{K_{rwb}}{\Delta x_{wb}} \quad \left(\frac{\partial P}{\partial x} \right)_b = \frac{q_t}{1.127 K_{abs} A_{n_{tb}}} = GPO \quad (3.32)$$

$$\begin{array}{l} \text{Boundary} \\ \text{conditions} \\ (i = N_x) \end{array} \quad \left(\begin{array}{l} S_b = \frac{3}{2} S_{N_x} - \frac{1}{2} S_{N_x-1} \\ p_b = p_{br} \\ T_b = \frac{3}{2} T_{N_x} - \frac{1}{2} T_{N_x-1} \end{array} \right) \quad \left(\begin{array}{l} K_{rob} = K_{ro} (1 - S_b) \\ K_{rwb} = K_{rw} (S_b) \\ \rho_{ob} = \rho_o(p_b, T_b) \\ \mu_{wb} = \mu_w(p_b, T_b) \end{array} \right)$$

By substituting this term into the equation below (discretized pressure equation for the last cell),

$$\begin{aligned} & \frac{\rho_o \frac{K_o}{\Delta x} \left(\frac{\partial P}{\partial x} \right)_{br}^{n+1} - \rho_o \frac{K_o}{\Delta x} \left(\frac{\partial P}{\partial x} \right)_{i-\frac{1}{2}}^{n+1}}{6.328 \Delta t} + \frac{1}{S_i^{n+1}} \left(\rho_{opi}^{n+1} p_i^{n+1} - p_i^n \right) + \frac{1}{S_i^{n+1}} \left(\rho_{oti}^{n+1} T_i^{n+1} - T_i^n \right) - \rho_{oi} S_i^{n+1} - S_i^n \\ & \quad (3.33) \end{aligned}$$

And noting that the generation and consumption term in the main equation 3.21 is replaced by the effect of the boundary condition, we have the equation below for $i = N_x$,

$$D^{n+1}$$

$$F(x) = Ax \quad B = 0 \quad (3.35)$$

Jacobian. As the number of equations and unknowns, N_x ; increases, so do the number of elements in the Jacobian.

The convergence of Newton's method is quadratic when the Jacobian matrix is non-singular and the initial guess is close enough.

;1614 985.se enough.

3.4 Jacobian Matrix Definition for Mass Balance Equation

In order to solve the nonlinear system resulted from applying fully implicit method to oil mass conservation equation (oil pressure equation), the Jacobian is defined as follow,

For internal equations ($i = 2; :::; N_x$

$$\frac{F}{R_1} = \frac{1}{2} \frac{n+1}{o_{i+1}} D_{oi+1}^{n+1} P_{i+1}^{n+1} P_i^{n+1} + D_{oi+1}^{n+1} \quad (3.44)$$

$$\begin{aligned} \frac{\partial F}{\partial P} = & \frac{1}{2} D_{oi}^{n+1} P_i^{n+1} + \frac{1}{6:328} \frac{S_i^{n+1}}{t} \\ & + \frac{1}{2} D_{oi}^{n+1} P_i^{n+1} + \frac{1}{6:328} \frac{S_i^{n+1}}{t} P_i^{n+1} + \frac{3}{2} D_{oi}^{n+1} P_i^{n+1} \times GPO \\ & + \frac{1}{6:328} \frac{S_i^{n+1}}{t} P_i^n + \frac{1}{6:328} \frac{S_i^{n+1}}{t} T_i^{n+1} T_i^n \\ & + \frac{1}{6:328} \frac{S_i^{n+1}}{t} S_i^n \end{aligned} \quad (3.45)$$

where

Derivatives of oil and water densities and viscosities to pressure and temperature are defined using the equations 2.9 - 2.12.

3.5 Well Coupling

The right boundary well pressure equation is

$$q_t = \frac{1:127K_{abs}A_n}{x=2} \left[\frac{K_{rw}}{w(P_{wr})} + \frac{K_{ro}}{o(P_{wr})} \right] (P_{wr} - P_B) \quad (3.49)$$

3.6 Energy Balance Equation Discretization

The energy equation 2.40 in oil field units (refer to nomenclature) is rewritten as

$$\begin{aligned} \frac{\partial}{\partial x} \left(24K_H \frac{\partial T}{\partial x} \right) + 6:328 \frac{\partial}{\partial x} \left(f \left(o \frac{K_o}{o} H_o + w \frac{K_w}{w} H_w \right) \frac{\partial P}{\partial x} g \right) + \frac{e}{V_b} \\ = \frac{\partial}{\partial t} \left(f \left(o S_o U_o + w S_w U_w \right) + (1 - f) U_r g \right) \end{aligned} \quad (3.50)$$

$$\begin{aligned} H_o &= H_o^{ref} + C_{Po} (T - T_{ref}) &= o; w \\ U_o &= U_o^{ref} + C_{Vo} (T - T_{ref}) &= o; w; r \end{aligned} \quad (3.51)$$

A fully implicit central finite difference scheme is used to discretize the equation. First, the equation is expanded by substituting equation 3.51 and then new equation is shortened by defining some parameters,

$$\begin{aligned} \frac{\partial}{\partial x} \left(24K_H \frac{\partial T}{\partial x} \right) + 6:328 \frac{\partial}{\partial x} \left(f \left(o \frac{K_o}{o} (H_o^{ref} + C_{Po}(T - T_{ref})) \right) \frac{\partial P}{\partial x} g \right) \\ + 6:328 \frac{\partial}{\partial x} \left(f \left(w \frac{K_w}{w} (H_w^{ref} + C_{Pw}(T - T_{ref})) \right) \frac{\partial P}{\partial x} g \right) + \frac{e}{V_b} = \\ \frac{\partial}{\partial t} \left(f \left(o S_o (U_o^{ref} + C_{Vo}(T - T_{ref})) + w S_w (U_w^{ref} + C_{Vw}(T - T_{ref})) \right) \right. \\ \left. + (1 - f) (U_r^{ref} + C_{Vr}(T - T_{ref})) g \right) \end{aligned} \quad (3.52)$$

By defining,

$$HR_o = o \frac{K_o}{o}$$

$$\frac{\partial}{\partial x} \left(24K_H \frac{\partial T}{\partial x} \right) + 6.328 \frac{\partial}{\partial x} \left((HR + HB:T) \frac{\partial P}{\partial x} \right) + \frac{P}{V_b} e = \frac{\partial}{\partial t} (UR + UB - T) \quad (3.57)$$

3.6.1 Discretization of Right Hand Side of the Energy Equation 3.57

Replace the derivatives as follow

$$\begin{aligned} \frac{\partial}{\partial t} (UR + UB:T) &= \frac{1}{t} f (UR + UB:T)^{n+1} - (UR + UB:T)^n g \\ &= \left(\frac{UR^{n+1} - UR^n}{t} \right) + \left(\frac{UB^{n+1} - UB^n}{t} \right) : T^{n+1} - \left(\frac{UB^n}{t} \right) T^n \end{aligned} \quad (3.58)$$

3.6.2 Discretization of Left Hand Side of Energy Equation 3.57 for Middle Cells ($i = 2; \dots; N_x$)

This side of the equation is divided into two terms; conductive and convective heat transfer. Conduction is the transfer of heat energy by diffusion due to the temperature gradient. In this project, conduction takes place in both rock and fluids. While convective heat transfer takes place through advection mostly, in which heat is transferred by the motion of currents in the fluid.

3.6.2.1 Conduction Term

In this dissertation, conductive heat is considered to transfer in two dimensions in order to model heat loss to adjacent strata [11]. Figure 3.4 shows the schematic diagram of the model.

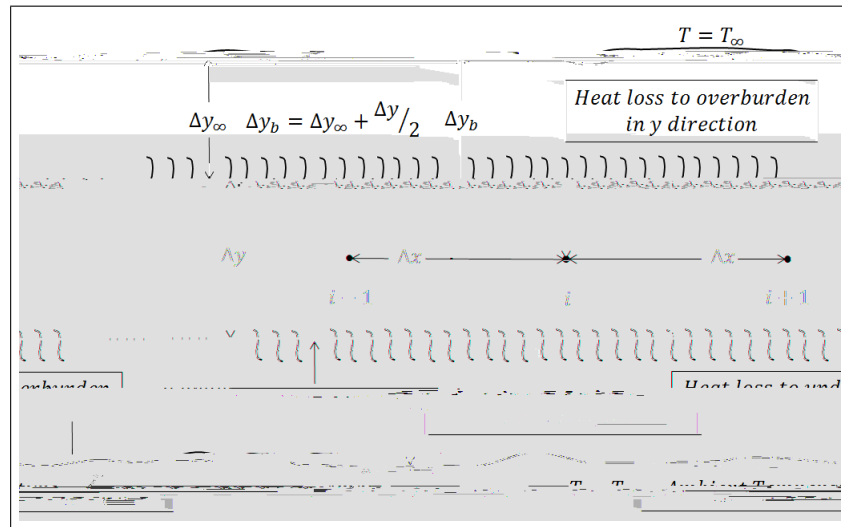


Figure 3.4: Heat transfer in the x and y direction by conduction

Using a central difference scheme with equal spacing in the x direction and unequal spacing in the y direction results in

$$\begin{aligned}
 \frac{\partial}{\partial x} \left(24K_H \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(24K_r \frac{\partial T}{\partial y} \right) = \\
 \frac{(24K_H \frac{\partial T}{\partial x})_{i+\frac{1}{2}}^{n+1} - (24K_H \frac{\partial T}{\partial x})_{i-\frac{1}{2}}^{n+1}}{x} + \frac{(24K_r \frac{\partial T}{\partial y})_{UP}^{n+1} - (24K_r \frac{\partial T}{\partial y})_{Down}^{n+1}}{y_1 + \frac{y}{2}} = \quad (3.59) \\
 \left(\frac{24K_H}{x^2} \right) T_{i-1}^{n+1} - \left(\frac{48K_H}{x^2} + \frac{48K_r}{y_b^2} \right) T_i^{n+1} + \left(\frac{24K_H}{x^2} \right) T_{i+1}^{n+1} + \left(\frac{48K_r}{y_b^2} \right) T_1
 \end{aligned}$$

where $y_b = y_1 + \frac{y}{2}$.

Convection term is sorted out as

$$\begin{aligned} \frac{\partial}{\partial x} \left((HR + HB) T \right) \frac{\partial P}{\partial x} = & \frac{6:328}{x} (HR_{i+1}^{n+1} : GP_{i+1}^{n+1} - HR_i^{n+1} : GP_i^{n+1}) \frac{6:328}{2x} (HB_i^{n+1} : GP_i^{n+1}) T_i^{n+1} + \\ & \frac{6:328}{2x} (HB_{i+1}^{n+1} : GP_{i+1}^{n+1} - HB_i^{n+1} : GP_i^{n+1}) T_i^{n+1} + \frac{6:328}{2x} (HB_{i+1}^{n+1} : GP_{i+1}^{n+1}) T_i^{n+1} \end{aligned} \quad (3.61)$$

By combining equations 3.57, 3.58, 3.59 and 3.61 together, the general energy equation for middle cells is obtained as($i = 2; \dots; N_x$);

$$\begin{aligned} f \frac{24K_H}{x^2} \frac{6:328}{2x} (HB_i^{n+1} : GP_i^{n+1}) g T_i^{n+1} f \frac{24K_H}{x^2} \frac{6:328}{2x} (HB_i^{n+1} : GP_i^{n+1}) g T_i^{n+1} \\ + f \frac{48K_H}{x^2} \frac{48k_r}{y_b^2} + \frac{6:328}{2x} (HB_{i+1}^{n+1} : GP_{i+1}^{n+1} - HB_i^{n+1} : GP_i^{n+1}) \frac{UB_i^{n+1}}{t} g T_i^{n+1} \\ + f \frac{24K_H}{x^2} + \frac{6:328}{2x} (HB_{i+1}^{n+1} : GP_{i+1}^{n+1}) g T_{i+1}^{n+1} = \left(\frac{UR_i^{n+1}}{t} - \frac{UR_i^n}{t} \right) \\ \left(\frac{UB_i^n}{t} \right) T_i^n - \left(\frac{48K_r}{y_b^2} \right) T_1 - \frac{6:328}{x} (HR_{i+1}^{n+1} : GP_{i+1}^{n+1} - HR_i^{n+1} : GP_i^{n+1}) \end{aligned} \quad (3.62)$$

3.6.3 Calculations for the Left Boundary Cell ($i = 1$)

For this cell, the conduction and convection terms are treated differently because of the effect of boundary conditions.

3.6.3.1 Conduction Term

Applying a central difference scheme to the conduction term in the x direction (with unequal spacing) gives;

$$\frac{\partial}{\partial x} \left(24K_H \frac{\partial T}{\partial x} \right) = \frac{24K_H \left(\frac{T_{i+1}^{n+1} - T_i^n}{x} \right) - 24K_H \left(\frac{T_i^{n+1} - T_{inj}}{x=2} \right)}{\left(\frac{3}{2}x \right)=2} \quad (3.63)$$

and in the y direction

$$\frac{\partial}{\partial y} \left(24K_r \frac{\partial T}{\partial y} \right) = \frac{24K_r \left(\frac{T_1 - T_i^{n+1}}{y_1 + y=2} \right) - 24K_r \left(\frac{T_i^{n+1} - T_1}{y_1 + y=2} \right)}{\left(y_1 + y=2 \right)} \quad (3.64)$$

By adding these two together and rearranging the terms, the conduction term is written as

$$f_i \left(\frac{\partial}{\partial x} (24K_H \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (24K_r \frac{\partial T}{\partial y}) \right) = \frac{\partial}{\partial x} (24K_H \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (24K_r \frac{\partial T}{\partial y}) =$$

$$\left(\frac{96K_H}{x^2} - \frac{48K_r}{y_b^2} \right) T_i^{n+1} + \left(\frac{32K_H}{x^2} \right) T_{i+1}^{n+1} + \left(\frac{64K_H}{x^2} \right) T_{inj} + \left(\frac{48K_r}{y_b^2} \right) T_1$$
(3.65)

3.6.3.2 Convection Term

By expanding the convection term on the first cell and substituting the injection temperature,

$$\frac{\partial}{\partial x} ((H_R + H_B) T) \frac{\partial P}{\partial x} =$$

$$\frac{6:328}{x} f_{H_R:GP} + H_B:GP:T g_{i+\frac{1}{2}}^{n+1} - \frac{6:328}{x} f_{H_R:GP} + H_B:GP:T g_b^{n+1} =$$

$$\left(\frac{3:164}{x} \right) (H_B^{n+1} : GP_{i+1}^{n+1}) (T_i^{n+1} + T_{i+1}^{n+1}) + \left(\frac{6:328}{x} \right) f_{H_R:GP} + H_B:GP:T g_{i+1}^{n+1} - H_R_i^{n+1} : GP_i^{n+1} g$$

$$- \left(\frac{6:328}{x} \right) (H_B_i^{n+1} : GP_i^n) T_{inj}$$
(3.66)

$$\begin{aligned}
6:328 \frac{@}{@x} ((HR + HB:T) \frac{@^P}{@x}) &= \frac{6:328}{x} (HR_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HR_i^{n+1} : GP_i^{n+1}) \\
+ (\frac{3:164}{x}) (3: HB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HB_i^{n+1} : GP_i^{n+1}) : T_i^{n+1} \\
(\frac{3:164}{x}) (HB_{i+1}^{n+1} : GP_{i+1}^{n+1} + HB_i^{n+1} : GP_i^{n+1}) : T_i^{n+1}
\end{aligned}$$

Based on above definitions and equation 3.62, the Jacobian calculations for $i = 2; \dots; N_x - 1$ give

$$\frac{\partial F}{\partial T_1} = \left(\frac{24K_H}{x^2} - \frac{3 \cdot 164}{x} (HB_i^{n+1} : GP_i^{n+1}) - 3 \cdot 164 \cdot x : DHR_i^{n+1} : GP_i^{n+1} \right. \\ \left. - 3 \cdot 164 \left(\frac{x}{2} \right) : DHB_i^{n+1} : GP_i^{n+1} : (T_i^{n+1} + T_{i-1}^{n+1}) \right)$$

$$\begin{aligned}
\frac{@F}{@Ti} = & f\left(\frac{3:164}{x}\right)(3HB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HB_i^{n+1} : GP_i^{n+1}) \quad \left(\frac{36K_r}{x: x_1}\right) \quad \left(\frac{48K_r}{y_b^2}\right) \quad \frac{UB_i^{n+1}}{t}g \\
& 3:164\left(\frac{x}{2}\right)(3DHB_{i+1}^{n+1} : GP_{i+1}^{n+1} + DHB_i^{n+1} : GP_i^{n+1}):T_i^{n+1} \\
& + f 3:164\left(\frac{x}{2}\right)(9DHB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad DHB_i^{n+1} : GP_i^{n+1}) \quad \frac{DUB_i^{n+1}}{t}g:T_i^{n+1} \\
& + 3:164: x:(3DHR_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad DHR_i^{n+1} : GP_i^{n+1}) \quad \frac{DUR_i^{n+1}}{t}
\end{aligned} \tag{3.82}$$

3.7 Summary of The First Model

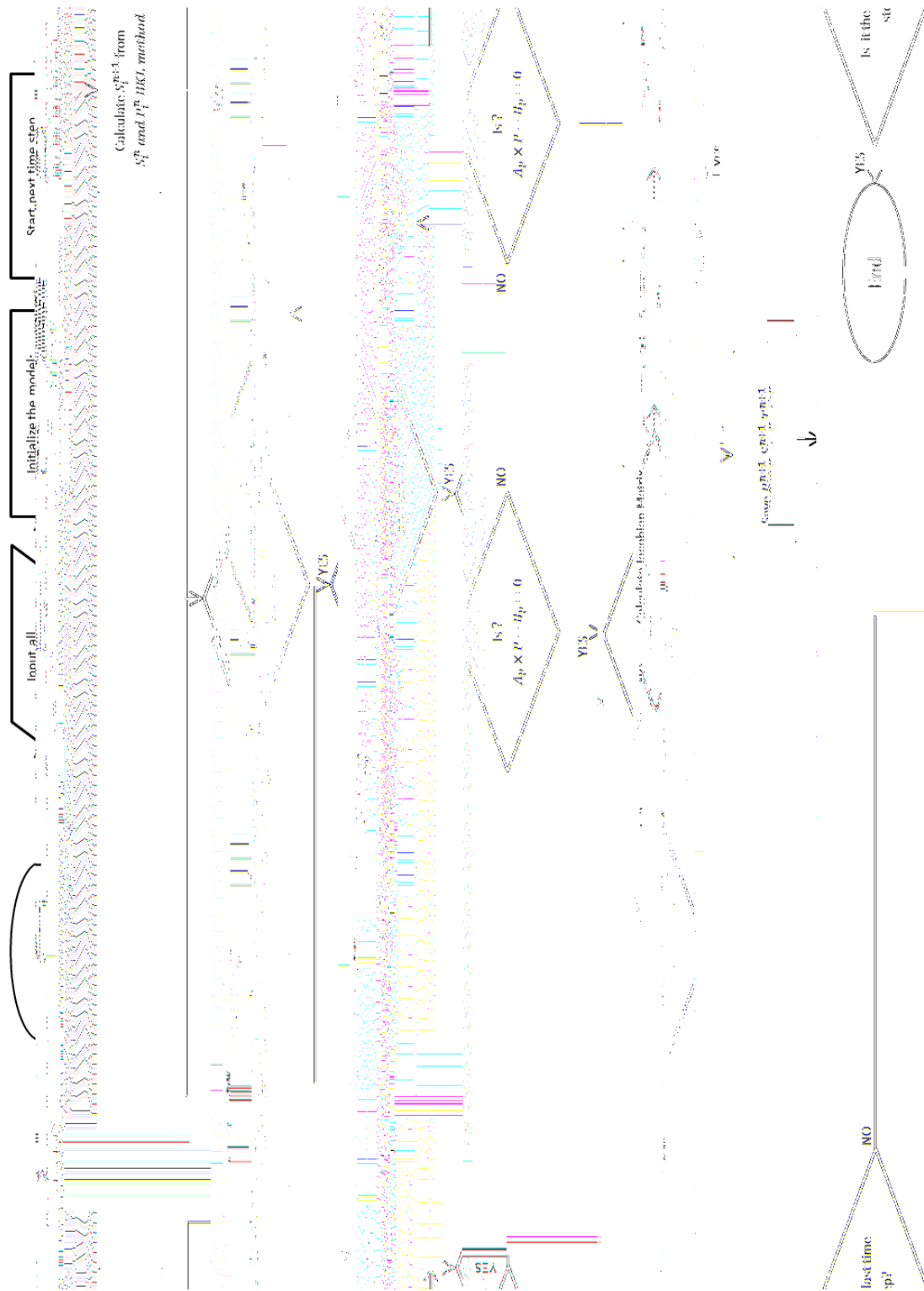


Figure 3.5: The First Model Calculations Flow Chart

4 Second Model

In this model, the pressure will be solved implicitly and after finding the pressure solution, saturation values can be determined explicitly. This technique is called IMPES and is much used in the oil industry [8]. During one time step the results of IMPES are used in the temperature equation which is solved fully implicitly, and finally there will be a coupling between the IMPES technique and the fully implicit temperature equation in order to find the final pressure, saturation and temperature distribution results.

4.1 IMPES Technique

In the oil and water system, the general 3D equations for oil and water are

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{K_o}{\mu_o} \frac{\partial P_o}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{K_o}{\mu_o} \frac{\partial P_o}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{K_o}{\mu_o} \frac{\partial P_o}{\partial z} \right) &= \frac{1}{\Delta t} \left(\frac{\partial S_o}{\partial t} \right) \\ \frac{\partial}{\partial x} \left(\frac{K_w}{\mu_w} \frac{\partial P_w}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{K_w}{\mu_w} \frac{\partial P_w}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{K_w}{\mu_w} \frac{\partial P_w}{\partial z} \right) &= \frac{1}{\Delta t} \left(\frac{\partial S_w}{\partial t} \right) \end{aligned} \quad (4.1)$$

By considering the assumptions made in section 2.1, and considering that generation and consumption terms are replaced by boundary conditions and expansion of right hand side of the equations, we have

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{K_o}{\mu_o} \frac{\partial P_o}{\partial x} \right) &= \frac{1}{\Delta t} \left((1 - S_o) \frac{\partial P_o}{\partial t} + (1 - S_o) \frac{\partial T}{\partial t} \right) \\ \frac{\partial}{\partial x} \left(\frac{K_w}{\mu_w} \frac{\partial P_w}{\partial x} \right) &= \frac{1}{\Delta t} \left(S_o \frac{\partial P_o}{\partial t} + S_o \frac{\partial T}{\partial t} \right) \end{aligned} \quad (4.2)$$

Divide oil pressure equation by μ_o and water pressure equation by μ_w , then add them together, and rearrange the final equation to find the discretized pressure equation for internal cells ($i = 2; \dots; N_x - 1$), giving

$$\begin{aligned} \frac{1}{\mu_o} \frac{\partial}{\partial x} \left(\frac{K_o}{\mu_o} \frac{\partial P_o}{\partial x} \right) + \frac{1}{\mu_w} \frac{\partial}{\partial x} \left(\frac{K_w}{\mu_w} \frac{\partial P_w}{\partial x} \right) &= \\ \frac{1}{\Delta t} \left((1 - S_o) \frac{\partial P_o}{\partial t} + S_o \frac{\partial P_o}{\partial t} \right) + \frac{1}{\Delta t} \left((1 - S_o) \frac{\partial T}{\partial t} + S_o \frac{\partial T}{\partial t} \right) \end{aligned} \quad (4.3)$$

$$\begin{aligned}
& \left(\frac{D_{oi}^{n+1}}{oi} + \frac{D_{wi}^{n+1}}{wi} \right) P_i^{n+1} - f \frac{D_{oi}^{n+1}}{oi} + \frac{D_{wi}^{n+1}}{wi} + \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{opi}}{oi} \right) + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wpi}}{wi} \right) g P_i^{n+1} = \\
& \text{GPO: } x: \left(\frac{D_{oi+1}^{n+1}}{oi} + \frac{D_{wi+1}^{n+1}}{wi} \right) - f \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{opi}}{oi} \right) + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wpi}}{wi} \right) g P_i^n \\
& - f \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{oTi}}{oi} \right) + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wTi}}{wi} \right) g (T_i^{n+1} - T_i^n) \\
& - f \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{oTi}}{oi} \right) + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wTi}}{wi} \right) g (T_i^{n+1} - T_i^n)
\end{aligned} \tag{4.8}$$

The result of writing the pressure equation for all blocks is also a nonlinear system to solve, so the definition of the Jacobian for this system is required.

4.2 Jacobian Calculations for the Pressure Equation

The general equation 4.4 is a function of three variables; P_{i-1} , P_i and P_{i+1} , therefore the Jacobian matrix is defined as follow

$$\frac{\partial F}{\partial P_{i-1}} = \frac{1}{2} \left(\frac{D_{wi}^{n+1}}{wi} \frac{n+1}{wpi} + \frac{D_{oi}^{n+1}}{oi} \frac{n+1}{opi} \right) (P_{i-1}^{n+1} - P_i^{n+1}) + \left(\frac{D_{oi}^{n+1}}{oi} + \frac{D_{wi}^{n+1}}{wi} \right) \tag{4.9}$$

$$\begin{aligned}
\frac{\partial F}{\partial P_i} = & f D_{oi+1}^{n+1} \left(\frac{1}{2} \frac{n+1}{oi} \frac{opi+1}{opi} - \frac{0_{opi}}{2 \cdot oi} \right) + D_{wi+1}^{n+1} \left(\frac{1}{2} \frac{n+1}{wi} \frac{wpi+1}{wpi} - \frac{0_{wpi}}{2 \cdot wi} \right) g P_{i+1}^{n+1} \\
& - f \frac{D_{oi}^{n+1}}{oi} + \frac{D_{oi+1}^{n+1}}{oi+1} + \frac{D_{wi}^{n+1}}{wi} + \frac{D_{wi+1}^{n+1}}{wi+1} + \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{opi}}{oi} \right) + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wpi}}{wi} \right) g \\
& - f \frac{D_{oi}^{n+1}}{oi} \frac{n+1}{opi} + \frac{D_{oi+1}^{n+1}}{oi+1} \frac{n+1}{opi+1} + \frac{D_{wi}^{n+1}}{wi} \frac{n+1}{wpi} + \frac{D_{wi+1}^{n+1}}{wi+1} \frac{n+1}{wpi+1} \\
& - (D_{oi}^{n+1} + D_{oi+1}^{n+1}) \left(\frac{0_{opi}}{oi} \right) - (D_{wi}^{n+1} + D_{wi+1}^{n+1}) \left(\frac{0_{wpi}}{wi} \right) + \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{oppi}}{oi} \right) \left(\frac{0_{opi}}{oi} \right)^2 \\
& + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wppi}}{wi} \right) \left(\frac{0_{wpi}}{wi} \right)^2 g P_i + f D_{oi}^{n+1} \left(\frac{1}{2} \frac{n+1}{oi} \frac{opi}{opi} - \frac{0_{opi}}{2 \cdot oi} \right) + D_{wi}^{n+1} \left(\frac{1}{2} \frac{n+1}{wi} \frac{wpi}{wpi} - \frac{0_{wpi}}{2 \cdot wi} \right) g P_{i-1}^{n+1} \\
& + f \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{oppi}}{oi} \right) \left(\frac{0_{opi}}{oi} \right)^2 + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wppi}}{wi} \right) \left(\frac{0_{wpi}}{wi} \right)^2 g P_i^n \\
& - f \frac{(1 - S_i^n)}{6:328 \cdot t} \left(\frac{0_{opTi}}{oi} \right) \left(\frac{0_{opi}}{oi} \right) \left(\frac{0_{oTi}}{oi} \right) + \frac{S_i^n}{6:328 \cdot t} \left(\frac{0_{wpTi}}{wi} \right) \left(\frac{0_{wpi}}{wi} \right) \left(\frac{0_{wTi}}{wi} \right) g (T_i^{n+1} - T_i^n)
\end{aligned} \tag{4.10}$$

$$\frac{\partial F}{\partial P_{i+1}} = \frac{1}{2} \left(\frac{D_{wi+1}^{n+1}}{wi} \frac{n+1}{wpi+1} + \frac{D_{oi+1}^{n+1}}{oi} \frac{n+1}{opi+1} \right) (P_{i+1}^{n+1} - P_i^{n+1}) + \left(\frac{D_{oi+1}^{n+1}}{oi} + \frac{D_{wi+1}^{n+1}}{wi} \right) \tag{4.11}$$

The first cell ($i = 1$) pressure equation depends on two variables; p_i and p_{i+1} , so

$$\begin{aligned} \frac{\partial F}{\partial P} = & f D_{oi+1}^{n+1} \left(\frac{1}{2} \frac{p_{oi+1}^{n+1}}{p_{oi}} - \frac{p_{oi}^0}{2} \right) + D_{wi+1}^{n+1} \left(\frac{1}{2} \frac{p_{wpi+1}^{n+1}}{p_{wi}} - \frac{p_{wpi}^0}{2} \right) g(p_{i+1}^{n+1} - p_i^{n+1}) \\ & f \frac{(1 - S_i^n)}{6.328 \cdot t} \cdot \frac{p_{oppi}^0 p_{oi} (p_{oi}^0)^2}{p_{oi}^2} + \frac{S_i^n}{6.328 \cdot t} \cdot \frac{p_{wppi}^0 p_{wi} (p_{wpi}^0)^2}{p_{wi}^2} g(p_i^{n+1} - p_i^n) \\ & f \frac{D_{oi+1}^{n+1}}{p_{oi}} + \frac{D_{wi+1}^{n+1}}{p_{wi}} + \frac{(1 - S_i^n)}{6.328 \cdot t} \cdot \frac{p_{oi}^0}{p_{oi}} + \frac{S_i^n}{6.328 \cdot t} \cdot \frac{p_{wpi}^0}{p_{wi}} g \\ & + \text{GPI: } x: f D_{oi}^{n+1} : \left(\frac{3}{2} \frac{p_{oi}^{n+1}}{p_{oi}} \right) \end{aligned}$$

4.3 Saturation Calculations

Oil and water saturations are evaluated explicitly by using the results of the fully implicit pressure equation. The following equations are applied to find the saturations

$$\begin{aligned} \frac{\partial}{\partial x} \left(\rho_o \frac{K_o}{\mu_o} \frac{\partial P}{\partial x} \right) &= \frac{1}{6.328} \frac{\partial}{\partial t} (\rho_o S_o) \\ \frac{\partial}{\partial x} \left(\rho_w \frac{K_w}{\mu_w} \frac{\partial P}{\partial x} \right) &= \frac{1}{6.328} \frac{\partial}{\partial t} (\rho_w S_w) \end{aligned} \quad (4.16)$$

These equations are discretized as

$$\begin{aligned} D_{oi+1}^{n+1} (P_{i+1}^{n+1} - P_i^{n+1}) - D_{oi}^{n+1} (P_i^{n+1} - P_{i-1}^{n+1}) &= \frac{1}{6.328} f_{oi}^{n+1} (S_{oi}^{n+1} - S_{oi}^n) g \\ D_{wi+1}^{n+1} (P_{i+1}^{n+1} - P_i^{n+1}) - D_{wi}^{n+1} (P_i^{n+1} - P_{i-1}^{n+1}) &= \frac{1}{6.328} f_{wi}^{n+1} (S_{wi}^{n+1} - S_{wi}^n) g \end{aligned} \quad (4.17)$$

Hence, oil and water saturations are calculated for internal cells ($i = 2; \dots; N_x$) from

$$\begin{aligned} S_{oi}^{n+1} &= \left(\frac{6.328}{f_{oi}^{n+1}} \right) \frac{1}{t} D_{oi+1}^{n+1} (P_{i+1}^{n+1} - (D_{oi}^{n+1} + D_{oi+1}^{n+1}) P_i^{n+1} + D_{oi}^{n+1} P_{i-1}^{n+1}) g + \left(\frac{n_{oi}}{n+1} \right) S_{oi}^n \\ S_{wi}^{n+1} &= \left(\frac{6.328}{f_{wi}^{n+1}} \right) \frac{1}{t} D_{wi+1}^{n+1} (P_{i+1}^{n+1} - (D_{wi}^{n+1} + D_{wi+1}^{n+1}) P_i^{n+1} + D_{wi}^{n+1} P_{i-1}^{n+1}) g + \left(\frac{n_{wi}}{n+1} \right) S_{wi}^n \end{aligned} \quad (4.18)$$

For the first cell ($i = 1$),

$$\begin{aligned} S_{oi}^{n+1} &= \left(\frac{6.328}{f_{oi}^{n+1}} \right) \frac{1}{t} D_{oi+1}^{n+1} (P_{i+1}^{n+1} + P_i^{n+1}) + GPl: x: D_{oi}^{n+1} g + \left(\frac{n_{oi}}{n+1} \right) S_{oi}^n \\ S_{wi}^{n+1} &= \left(\frac{6.328}{f_{wi}^{n+1}} \right) \frac{1}{t} D_{wi+1}^{n+1} (P_{i+1}^{n+1} + P_i^{n+1}) + GPl: x: D_{wi}^{n+1} g + \left(\frac{n_{wi}}{n+1} \right) S_{wi}^n \end{aligned} \quad (4.19)$$

and finally for the right boundary cell ($i = N_x$)

$$\begin{aligned} S_{oi}^{n+1} &= \left(\frac{6.328}{f_{oi}^{n+1}} \right) \frac{1}{t} D_{oi}^{n+1} (P_i^{n+1} - P_{i-1}^{n+1}) + GPO: x: D_{oi+1}^{n+1} g + \left(\frac{n_{oi}}{n+1} \right) S_{oi}^n \\ S_{wi}^{n+1} &= \left(\frac{6.328}{f_{wi}^{n+1}} \right) \frac{1}{t} D_{wi}^{n+1} (P_i^{n+1} - P_{i-1}^{n+1}) + GPO: x: D_{wi+1}^{n+1} g + \left(\frac{n_{wi}}{n+1} \right) S_{wi}^n \end{aligned} \quad (4.20)$$

This is the whole procedure of the IMPES method for this model.

In summary, in this model pressure and saturations are calculated from the IMPES technique, but a similar method (fully implicit method) with the first model is used to find the temperature distribution.

4.4 Summary of the Second Model

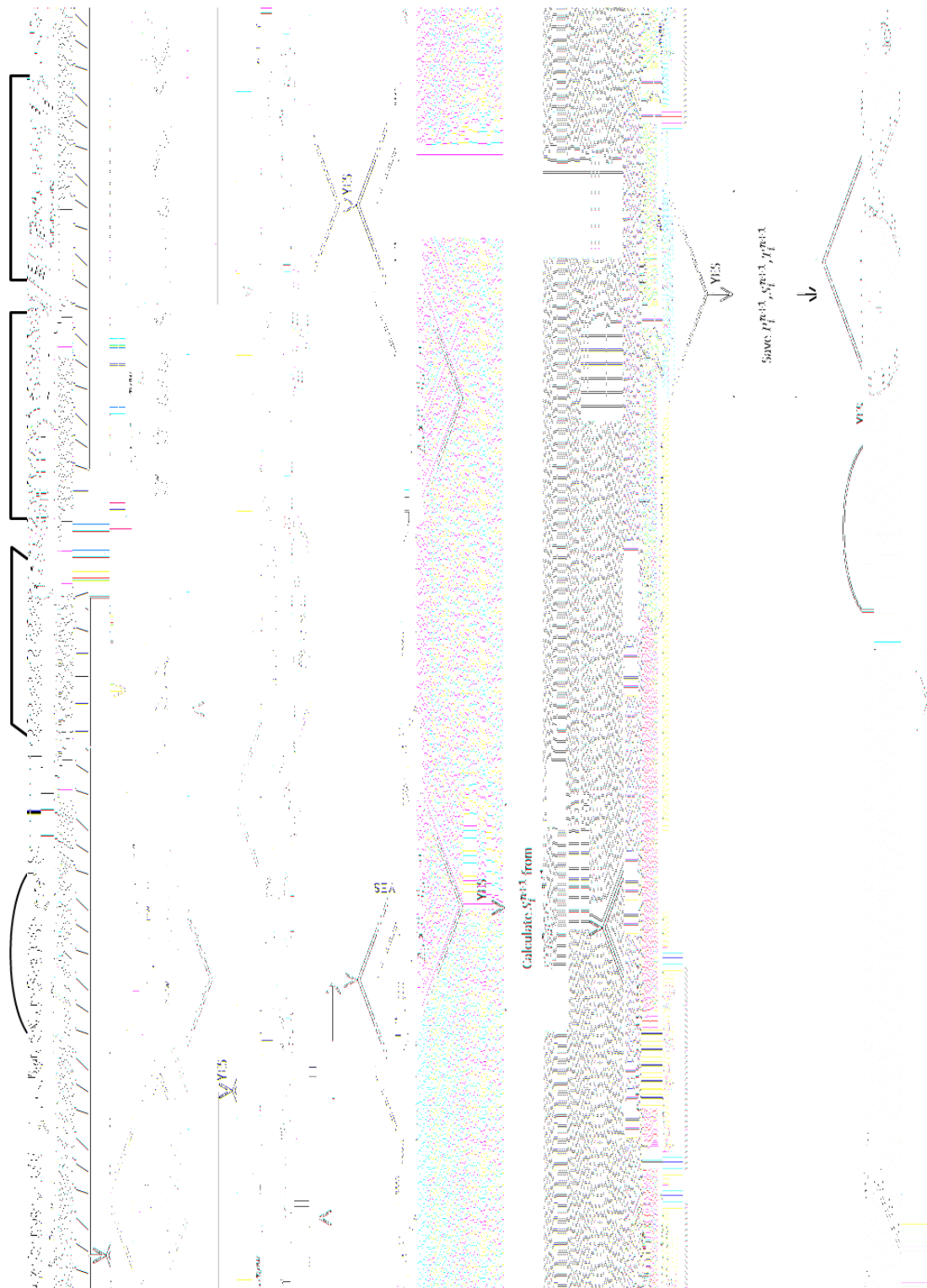


Figure 4.1: The Second Model Calculations Flow Chart

5 Results

It is interesting to see the results of the two different numerical techniques applied to a physical process and to see how choosing between these different techniques can change the results using the same inputs. Table 1 shows the values of the model parameters used in the two models.

5.1 First Model Results

In this model, the Buckley-Leverett equation and mass and energy balance equations are solved using the Lax-Wendro scheme and a fully implicit central schemes, respectively, in order to find saturation, pressure and temperature distribution in the one dimensional hot water model. The results for pressure, saturation and temperature profiles are shown in figure 5.1 using a step size of $\Delta x = 5(\text{ft})$ and a time step of $\Delta t = 0.025(\text{day})$. Distributions are plotted after 100, 250 and 400 days. It can be seen from figure 5.1 (the water saturation profile) that initially there is no steep front in the system but later, due to injection a shock (water front) is created which moves to the right in time. The pressure profile changes based on the water and oil properties and shock position. Surprisingly, there are no oscillations around the discontinuity (water saturation front) although the second order accurate Lax-Wendro scheme is used to solve Buckley-Leverett equation. Figure 5.2 shows that with higher number of divisions the front is steeper, as expected, but still no oscillation is observed around this steep front. This behavior might be related to using Lax-Wendro for unequal spacing and backward in space and forward in time schemes at the boundaries or front is not steep enough.

5.2 Second Model Results

The fully implicit pressure explicit saturation (IMPES) and fully implicit temperature tech-

5.3 Comparing Two Models

By looking at the results of the two models it is clear that both have similar trends for P, S and T. Whereas, by looking at the results more closely, it can be seen that the position of steep front in the first model lag behind its corresponding front position in second model, which could be a result of using the incompressible fluid assumption in the Buckley-leverett equation for the saturation profile. Consequently lower pressure values result in the first model (lower injected fluid lower pressure increase). Therefore, the second model using IMPES technique for solving pressure and saturation and fully implicit method for temperature is a better numerical model. The only problem is that it has the numerical instability in the IMPES technique which comes from using an explicit saturation calculation. So, to make the model more applicable, the stability limits must be considered. In the next section some sensitivity analysis is performed to find these limits.

5.4 Sensitivity Analysis

Sensitivity analysis is performed on the second model to investigate the influence of grid and

Table 1: Model Properties

Model properties	Value (Units)
Length (L)	150 (ft)
Width (dy)	20 (ft)
Height (dz)	30 (ft)
Distance to ambient temperature in x direction($4 x_1$)	100 (ft)
Distance to ambient temperature in y direction ($4 y_1$)	100 (ft)
Porosity (ϕ)	0.2
Absolute permeability (K_{abs})	3 (darcy)
Initial temperature (T_{init})	559.67 ($^{\circ}R$)
Initial pressure (P_{init})	4000 (psia)
Initial oil saturation ($S_{o init}$)	0.84
Total rate (q_t)	1 (bbl/day)
Reference temperature (T_{ref})	536.4 ($^{\circ}R$)
Ambient temperature (T_1)	559.67 ($^{\circ}R$)
Injection temperature (T_{inj})	800 ($^{\circ}R$)
Rock density (ρ_r)	145 (lb/ft ³)
Rock thermal conductivity (k_r)	0.9824 (Btu/ft.hr. $^{\circ}F$)
Average thermal conductivity, rock, oil and water (k_H)	0.4623 (Btu/ft.hr. $^{\circ}F$)
Oil specific gravity (S_{go})	0.9
Water specific heat capacity, constant pressure (C_{pw})	0.986 (Btu/lb. $^{\circ}F$)
Water specific heat capacity, constant volume (C_{vw})	0.932 (Btu/lb. $^{\circ}F$)
Rock specific heat capacity, constant volume (C_{vr})	0.22 (Btu/lb. $^{\circ}F$)

Table 2: Sensitivity Analysis on Nx (Nt=5000 , dt=0.01)

Nx	CPU Time (sec)	Stability Condition
5	108	Stable, very high numerical dispersion
10	156	Stable, high numerical dispersion
30	364	Stable, moderate numerical dispersion
60	712	Some oscillation, low numerical dispersion
150	2244	Unstable, very low numerical dispersion
300	5408	Unstable

Table 3: Sensitivity Analysis on dt (Nx=30 , Final time=50 Day)

Nx	CPU Time (sec)	Stability Condition
0.01	364	Stable
0.05	76	Stable
0.1	36	No oscillation, over-saturated
0.5	8	No oscillation, over-saturated
1	8	Some oscillation
2	8	More oscillation
5	{	Nearly singular matrix - no result

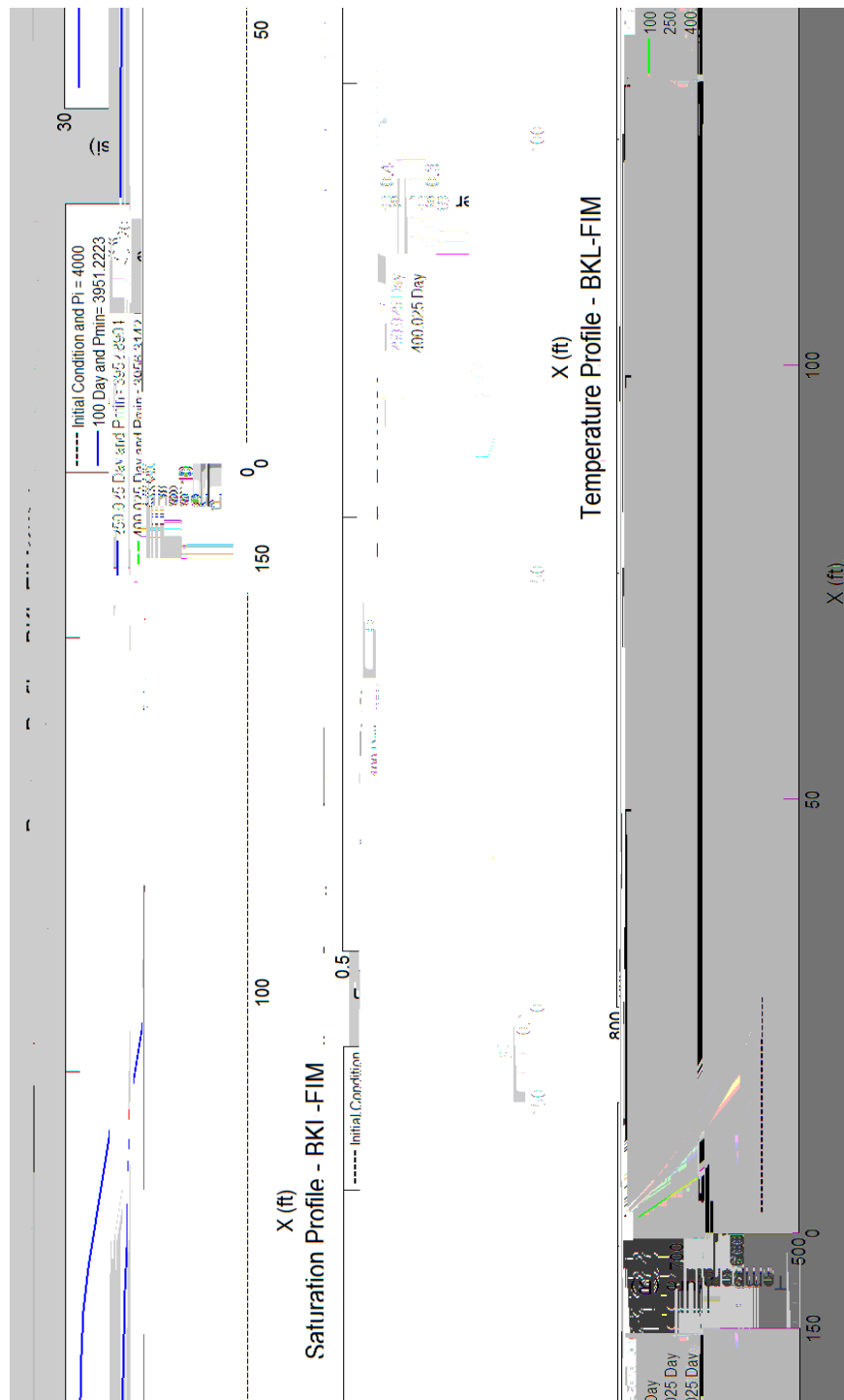
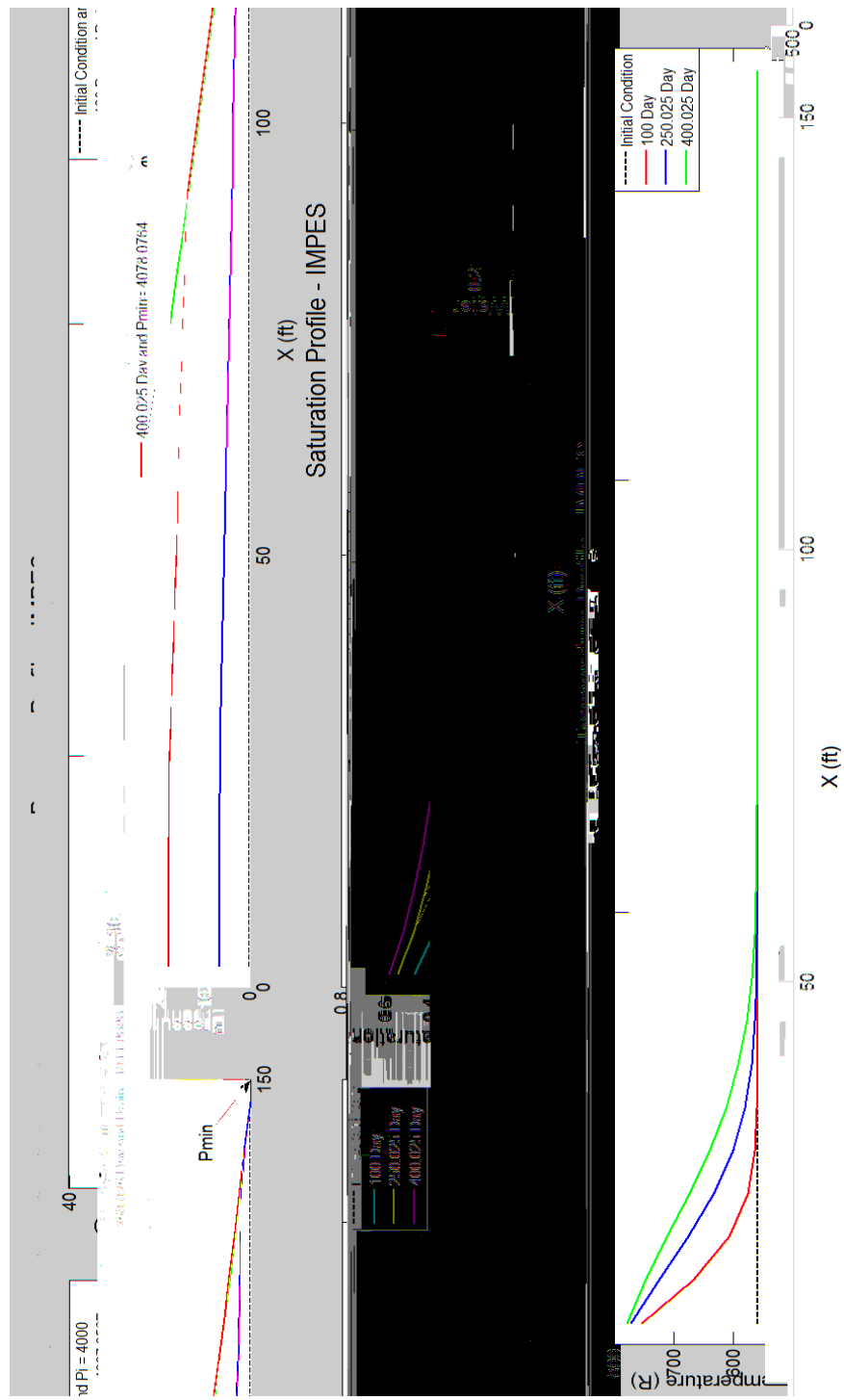


Figure 5.1: The First Model Pressure, Saturation and Temperature Results



Figure 5.2: The First Model Pressure, Saturation and Temperature Results , $N_x=300$



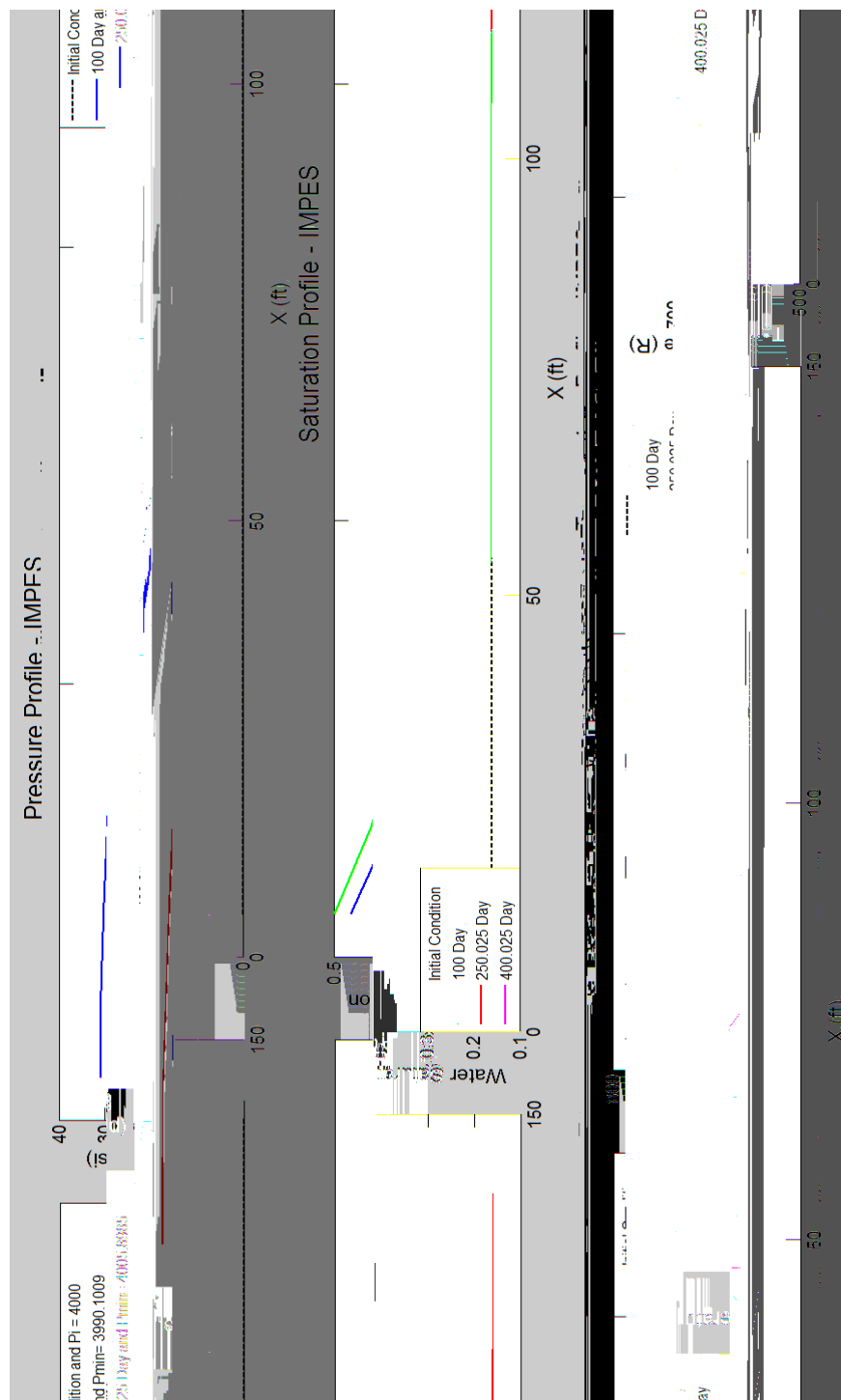
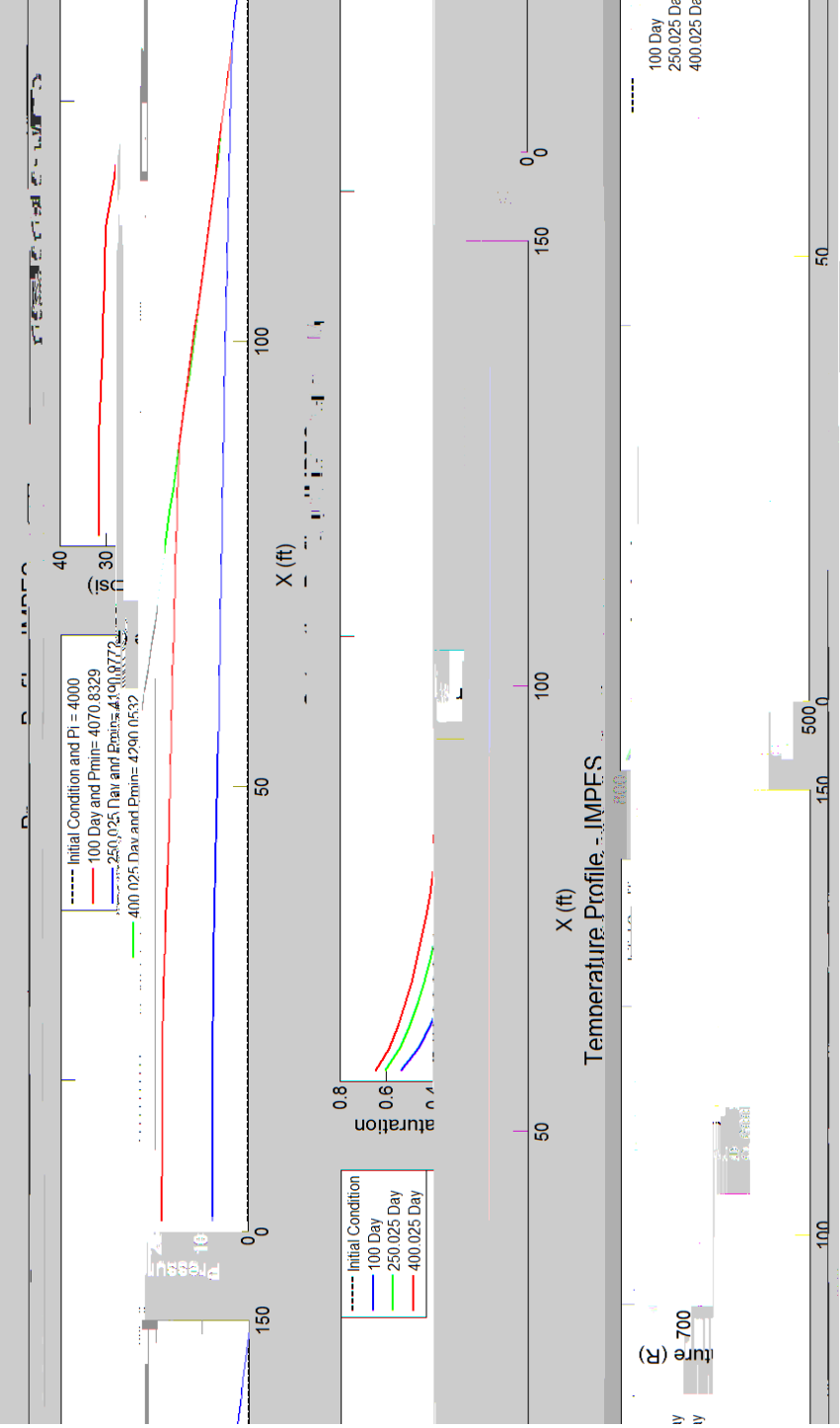


Figure 5.4: Pressure, Saturation and Temperature Profiles for the second model for $N_x=15$



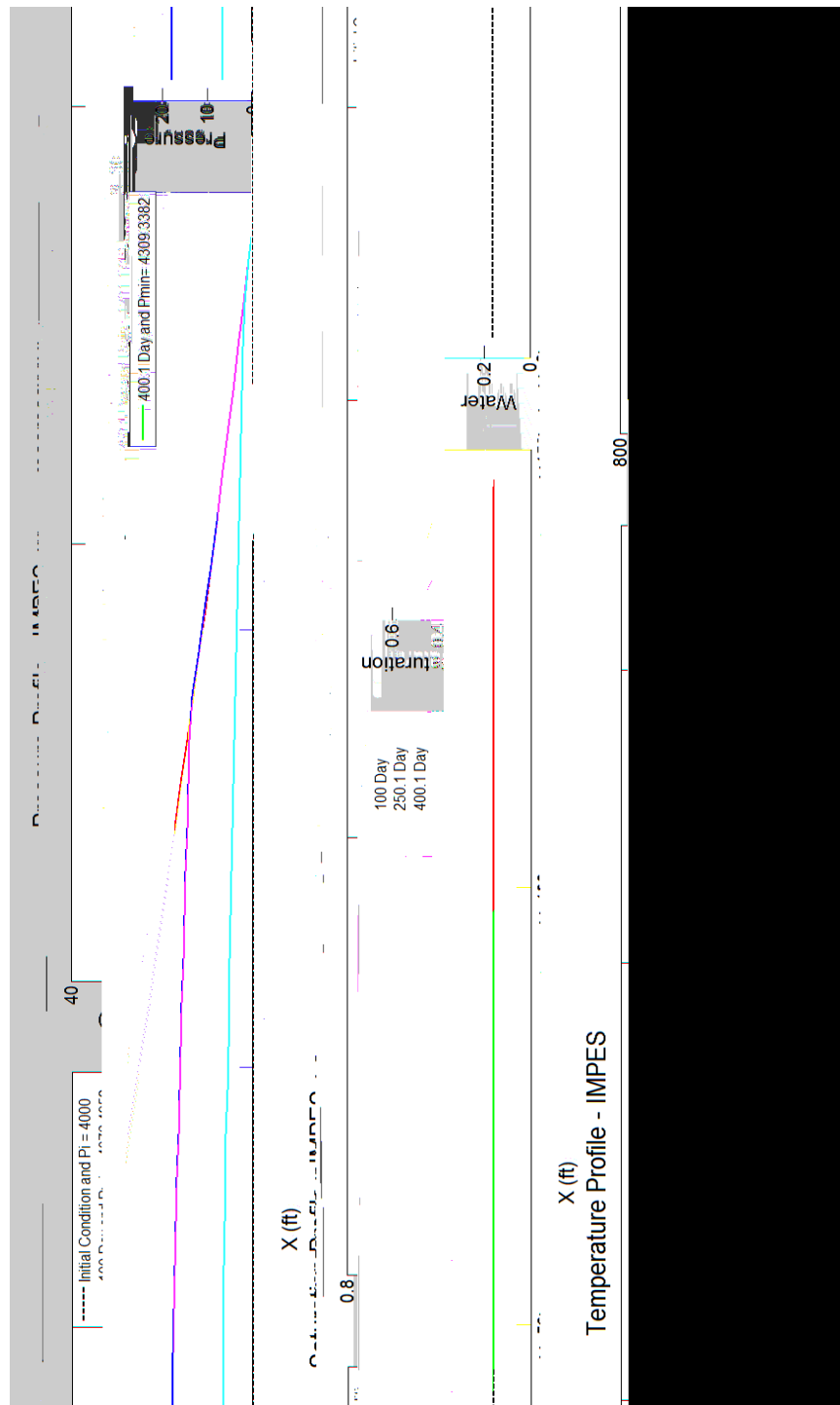


Figure 5.6: The Second Model Pressure, Saturation and Temperature Profiles (dt=0.1 Day)

6 Conclusion

In this dissertation two numerical models have been applied to a method of oil recovery, hot water injection. This physical process can be described using oil and water mass conservation equations, energy balance and Buckley-Leverett equations. Hot water injection is modeled by using these equations in order to find pressure, saturation and temperature profiles.

In first model Lax-Wendro and fully implicit scheme have been used to solve Buckley-Leverett, oil mass and energy balance equations respectively. The model may be less reliable since one of the basic assumptions of Buckley-Leverett equation is considering fluids as incompressible which is not a reasonable assumption in this study. It is worth noticing that

results with the result of this study. For instance, oil and water mass balance equations and energy balance equation can be modeled using fully implicit method for all of them and solve them simultaneously which is a well-known method but, it is really expensive technique in calculation.

Using different boundary conditions (well models) to investigate their effect on the saturation, pressure and temperature distributions in the system.

In first model, other numerical methods like Warming-Beam, Fromm, etc., can be used to find the saturation in Buckley-Leverett equation.

Nomenclature

P	Pressure (psia)
V	

Subscripts and Superscripts

	Phase index (o = oil; w = water)
w	Water phase
o	Oil phase
b	Boundary
c	Capillary pressure
abs	Absolute
p	Pore or Pressure
r	Relative or Rock
t	Total
prod	Production
inj	Injection
gen	Generation
cons	Consumption
V	Volume
or	Residual oil
wc	Connate water
av	Average
ref	Reference

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