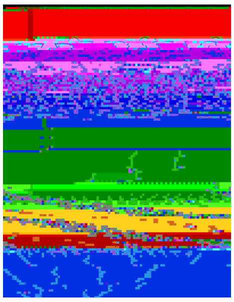

UNIVERSITY OF READING
Department of Mathematics, Meteorology & Physics

The Influence of the Agulhas leakage on the overturning circulation from momentum balances

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This dissertation is submitted to the Departments of Mathematics and Meteorology in partial fulfilment of the requirements for the degree of Master of Science

Declaration

I confirm that this is my own work, and the use of all material from other sources has been properly and fully acknowledged.

Signed: ....

Dated: ....

Acknowledgements

I would like to take this opportunity to thank all my family for their forever love and devotion, particularly my parents. I would not be in this position at this moment in time without them. A special thank you goes to my aunt and uncle for having me for the entire year and coping with me, especially during some intense academic times. I would like to thank my supervisor, Profesor Peter Jan van Leeuwen, for his

Abstract

We want to investigate the influence of Agulhas rings on the ocean circulation in the South Atlantic, and so on the Overturning Circulation. The Agulhas leakage between the South Atlantic and Indian Ocean is an interocean exchange transporting heat and salt from one basin to the other. The study incorporates use of momentum and vorticity balances in order to explain the impact of Agulhas rings on the Overturning Circulation. These rings are shed in the South Atlantic and alter the momentum balance in this ocean basin. The normal occurrence is geostrophic balance, but we find an additional advection contribution in the balance. Allowing Agulhas rings to enter the basin, it becomes evident that the vorticity balance changes in comparison to a closed basin where we have no rings shed. This directly infers that the rings do have a significant influence on the circulation in the South Atlantic, and thus potentially on the Overturning Circulation.

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1 Introduction

A. L. Gordon (1985) has shown that the Indian Ocean surface water is more than 5 C warmer and has a richer salt content than the South Atlantic, approximately 0.3-0.4 practical salt units (psu) higher. These Agulhas rings transport water from the Indian Ocean into the South Atlantic which can have a major influence on the Thermohaline Circulation.

We will study the interaction between the dynamics and thermodynamics of the ocean, by investigating the momentum and vorticity balances in a reduced gravity model of the South Atlantic. This will contribute to our understanding of the Thermohaline Circulation.

By integrating the momentum and vorticity equations over an ocean basin with either closed or open boundaries, we study the relation between the wind-stress input, via the dynamics, and the behaviour of the thermocline.

The idea is to systematically study momentum and vorticity balances over the South Atlantic basin. We start with a closed basin, then move onto an open basin that gradually becomes more complicated and will eventually build up to the full problem.

Firstly, we will look at the integrated momentum equations and vorticity equation solely on the closed basin, looking at the subtropic gyre contained within it. We expect to learn something about the balances that include the information about the height profile. We expect the vorticity balance can provide a greater understanding of the physical aspects of the ocean circulation. We will then run a numerical model of the closed basin circulation and try to confirm the findings from our analytical calculations of the momentum balances.

We will then apply the momentum equations to the same South Atlantic basin implementing different boundary conditions. These boundary conditions will create

Agulhas leakage is present in the basin. Using the numerical model, we aim to obtain a visual understanding of the Agulhas leakage in the ocean basin. We then have the ability to numerically calculate terms in the vorticity equation for both cases and distinguish a vorticity balance for both cases.

Studying the momentum equations and vorticity equation on the South Atlantic Ocean, using analytical and numerical methods, will give us an insight to what influence the Agulhas leakage has on the ocean basin as well as the global Overturning Circulation.

2 The Governing Equations

$$u_t + uu_x + vu_y - fv = -g'h_x + \frac{A}{h}\Big[(hu_x)_x + (hu_y)_y\Big] + \frac{\tau^{(x)}}{h} \qquad (7)$$

$$v_t + uv_x + vv_y + fu = -g'h_y + \frac{A}{h}\Big[(hv_x)_x + (hv_y)_y\Big]. \qquad (8)$$

Taking the curl of these equations, i.e. $\frac{\partial}{\partial x}(8) - \frac{\partial}{\partial y}(7)$ gives

$$\begin{aligned}\zeta_t + u\zeta_x + v\zeta_y + (f+\zeta)(u_x+v_y) + \beta v = & -\left(\frac{\tau^{(x)}}{h}\right)_y \\ & + A(\zeta_{xx} + \zeta_{yy}) + A\left[\left(\frac{h_x v_x}{h}\right)_x + \left(\frac{h_y v_y}{h}\right)_y - \left(\frac{h_x u_x}{h}\right)_x - \left(\frac{h_y u_y}{h}\right)_y\right].\end{aligned} \qquad (9)$$

We multiply (9) by h and use continuity (3), to give the Vorticity Equation (10)

$$\begin{aligned}(h\zeta)_t + (hu\zeta)_x + (hv\zeta)_y + h(f+\zeta)(u_x+v_y) + \beta hv = & -h\left(\frac{\tau^{(x)}}{h}\right)_y \\ & + hA(\zeta_{xx} + \zeta_{yy}) + hA\left[\left(\frac{h_x v_x}{h}\right)_x + \left(\frac{h_y v_y}{h}\right)_y - \left(\frac{h_x u_x}{h}\right)_x - \left(\frac{h_y u_y}{h}\right)_y\right].\end{aligned} \qquad (10)$$

In the South Atlantic Ocean, we know that rings are shed from the Agulhas leakage. The vorticity equation is useful, since we can measure the spin induced from these rings as well as other vortices that are formed. We aim to find out what effect they have on the vorticity equation. With analytical calculations and a numerical model, described in Section 3, we hope to find out what influence the Agulhas leakage has on the Overturning Circulation.

3 Numerical Model

Using the two layer model proposed earlier, we will be using a numerical model of the South Atlantic Ocean basin. This aims to provide a detailed picture and view of the circulation in the basin. Analytical calculations cannot provide us with detail of the circulation within the basin.

Although numerical modelling cannot output the real situation perfectly, they are increasingly becoming on par with them. As our understanding of these situations improve, they are beginning to play a more dominant role. Both time and space parameters are important in numerical modelling. We are able to calculate what time step is best for the model without it blowing up. Our analytical calculations are assumed to be in steady state which means there are no time variations. Using our numerical model, we are able to accomplish outputs that are based on time-averaged flow. These outputs provide us with a useful picture of what is occurring within the basin.

The form of the equations we are going to discretise are slightly different to the equations given in Section 2. Writing equations (1) and (2) as the equations given in (11) and (12) makes sure potential vorticity is conserved in the absence of friction.

The equations that we discretise in the numerical model are given by

$$u_t + \frac{1}{2}(u^2 + v^2)_x \quad (\quad + f)v = \quad g^0 h_x + A \quad u_{xx} + u_{yy} \quad + \frac{(x)}{h} \tag{11}$$

$$v_t + \frac{1}{2}(u^2 + v^2)_y + (\quad + f)u = \quad g^0 h_y + A \quad v_{xx} + v_{yy} \quad : \tag{12}$$

Note the different forms for friction. These are not identical but both have a sound physical basis. The details will not be discussed here. The analytical form for friction has its advantages for analytical derivations.

These equations are discretised using a Leapfrog scheme, with an initial Euler step. The Leapfrog scheme can develop an unphysical, numerical mode, that itself is stable but the solution is wrong. In order to damp this computational mode, we employ an Euler step every 16^{th} time-step. We discretise the equations on a 2-dimensional staggered grid known as the Arakawa-C grid. The boundary conditions given are no-slip boundary conditions in which u and v are both zero on all boundaries.

The time and space parameters used in the model are specifically chosen to guarantee convergence. The Courant-Friedrichs-Lewy (CFL) condition is a necessary condition which tells us the upper limit for a time-step to be used in the model.

The CFL condition is given by $t < \frac{x}{c}$, where t is the time-step, x is grid spacing and c is the velocity of the fastest wave, which is the reduced gravity wave here.

Our model has a grid spacing, $x = 1000$ m. This value is chosen to be eddy-permitting. We know for that for gravity waves, $c = \sqrt{g^0 h} = 3{:}16\ ms^{1}$. This gives us an upper bound for the time-step which is 316 s. In order to keep to this CFL condition, we take a time-step, $t = 225\ s$.

In terms of running the model, we look at the baroclinic Rossy waves travelling across the ocean basin. For these Rossby waves, $c = R_d^2$, where

The numerical model will be used to confirm findings generated by the momentum equations and to look at the vorticity within the basin. As well as numerically modelling the basin, we will use the model to numerically calculate terms in the vorticity equation, that are time-averaged and look at how it is balanced.

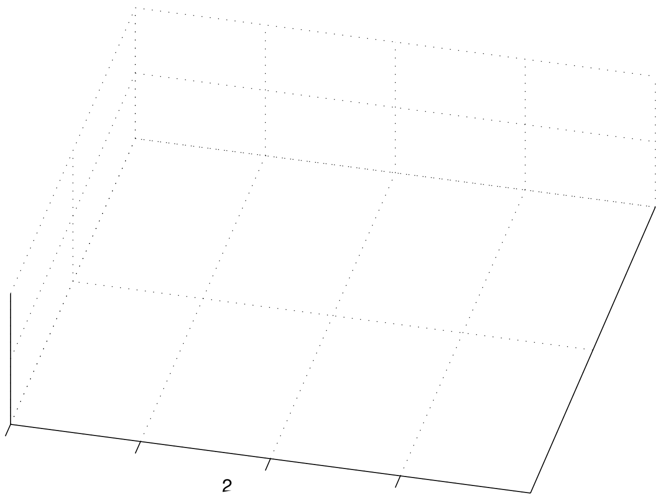


Figure 3: Plot showing output of model when run for 1000 days.

Figure 4: Plot of the island used in the numerical model.

$$\Rightarrow \int \left[huv\right]_{y=0}^{y=M} dx + \int \left[(hu^2) + \tfrac{1}{2}g'(h^2)\right]_{x=0}^{x=L} dy - \iint \left[fhv + A\nabla\cdot(h\nabla u) + \frac{\tau^{(x)}}{\rho} \right] dx\,dy = 0.$$

We know that $u = 0$ on the eastern and western boundaries and $v = 0$ on the northern and southern boundaries so the first and second terms vanish. We expect that these do vanish because they are the advection terms and on a closed basin we should find no advecting mass on the boundaries. Furthermore, since the basin is closed, we cannot introduce any new mass into the basin. This means that there can be no other advection so these terms should always remain zero.

So we are left with

$$\int \tfrac{1}{2}g'[h^2]_{x=0}^{x=L}\,dy - \iint \left[fhv + A\nabla\cdot(h\nabla u) + \frac{\tau^{(x)}}{\rho}\right] dx\,dy = 0.$$

Equation (4) allows us to introduce a transport stream function by $hu = -\psi_y$ and $hv = \psi_x$. Indeed, if substitute these into (4) gives us $-\psi_{xy} + \psi_{xy} = 0$, where we call ψ a stream function. Integrating the stream function in the zonal direction gives us

$$\int_0^L \psi_x\,dx = \int_0^L hv\,dx$$
$$\Rightarrow \psi(L) - \psi(0) = \int_0^L hv\,dx.$$

The stream function measures the transport. The amount of water flowing northward in the interior of the basin is equal to the amount flowing southward along the western boundary. This tells us that

$$\psi(L) - \psi(0) = \int_0^L hv\,dx = 0.$$

Using what we know about the stream function, we can now rewrite the Coriolis term as

$$\iint fhv\,dx\,dy = \iint f\psi_x\,dx\,dy = \int f[\psi]_{x=0}^{x=L}\,dy = 0.$$

We find that the Coriolis force has no effect on the closed basin in the zonal direction.

This now leaves us with

$$\int \tfrac{1}{2}g'[h^2]_{x=0}^{x=L}\,dy = \iint \left[A\nabla\cdot(h\nabla u) + \frac{\tau^{(x)}}{\rho}\right] dx\,dy.$$

The first term on the right hand side of the equation, the friction term, can be written as

$$\iint A \nabla \cdot (h \nabla u)\, dx\, dy = \iint A[(hu_x)_x + (hu_y)_y]\, dx\, dy$$
$$= A \int \left. hu_x \right|_{x=0}^{x=L} dy \quad A \int \left. hu_y \right|_{y=0}^{y=M} dx.$$

The second term on the right hand side is the wind stress. The wind stress is

Note that we are using the coordinate system such that water flowing eastwards has a positive value for u and is negative for water flowing westwards.

Using the total contribution which is just the friction term, we can find the pressure term. This means that

$$\int \tfrac{1}{2} g' [h^2]_{x=0}^{x=L}\, dy = -5000\, m^4 s^{-2}$$
$$\implies [h^2]_{x=0}^{x=L} = -0.25\, m^2.$$

Showing us that the difference here must be small. This tells us something about the height profile at each of the boundaries at $x = 0$ and $x = L$. Since the pressure term must balance the momentum equation in the zonal direction and the integral must be small, the depth of water must be similar at both boundaries.

If we take the integral, $\int \frac{1}{2} g' [h^2]_{x=0}^{x=L}\, dy$, the only way this can be small is if the integral evaluated at each limit equals each other. In other words

$$\int \tfrac{1}{2} g' [h^2]_{x=L}\, dy = \int \tfrac{1}{2} g' [h^2]_{x=0}\, dy.$$

So this tells us that $h(x=L) \approx h(x=0)$. The depths are about equal at both the east and west boundaries.

4.2 The Meridional Momentum Equation

We now look at the meridional momentum equation, equation (6). In a similar manner, we integrate over the closed basin Ω and apply Stoke's theorem to this equation. We get the following

$$\iint_\Omega \left[(huv)_x + (hv^2)_y + fhu + \tfrac{1}{2} g' (h^2)_y - A \nabla \cdot (h \nabla v) \right] dx\, dy = 0.$$

Both the advection terms are zero since we know that for a gyre in this closed basin $u = 0$ on the eastern and western boundaries and $v = 0$ on the northern and southern boundaries. As mentioned in the zonal momentum equation previously, we expect that the advection are zero on the closed basin. There is also no wind stress in this direction. We are left with

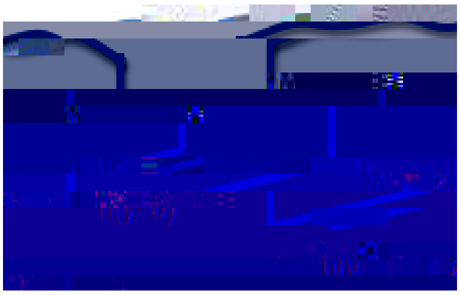
$$\iint_\Omega \left[fhu + \tfrac{1}{2} g' (h^2)_y - A \nabla \cdot (h \nabla v) \right] dx\, dy = 0.$$

We saw previously how we can write the friction term and we get the following

[(1)]T575 0 Td[.411.9552 Tf 1Td 176.6(anET q 1 0 0 1 242.178 235.887 cm []0 d 0 J 0.705 w 0 0 m 3.9 Tf 1Td 16 0 26F63 7.9701 Tf 242.178 228.522 Td [(2)]TJ/F66

[redacted] $=) \int \frac{1}{2} g^0 [h^2]_{y=M} > \int \frac{1}{2} g^0 [h^2]_{y=0}$.

[redacted] tion is that the water is deeper north of the basin and shallower south [redacted] See Figure 6 below.



[redacted] igure 6: Typical depth profile in the meridional direction.

[redacted] erical Model Verification

[redacted] l results of the momentum balances in the zonal and meridional tell [redacted] height profile of the basin using a two-layer model. We now wish to [redacted] esults using our numerical model.

[redacted] numerical model until a statistical equilibrium is reached allows us to [redacted] numerical results to our analytical calculations. This is achieved after [redacted] y 1000 days.

abile [redacted] lepths at the boundaries of the closed basin we fin.ep(prof9me)-205(the)-284(numerical) [redacted] he)-29(elevlatio,[)]TJ/F6611.9552Tf126.37810Td[e0 [redacted] he)-480(numerical)3475(model)-480(confirsl)-480(our)3475(analytical)-480(finninns.)6

[redacted]

4.4 Estimating the Vorticity Equation on the Closed Basin

$$! \quad h \frac{(x)}{h} \; y \; dx\,dy \qquad H \frac{(x)}{H \; M} LM = \frac{(x)}{} L.$$

The wind stress over the basin is has an approximate value of $^{(x)} = 0{:}1Pa$ and the u.9780

A method used to do this is called Reynolds Averaging. This is where turbulent fluctuations are separated from the mean flow. It lets us average terms in time.

In order to do this we write our variables as a time-averaged part and a time-varying part, for example

$$
\begin{aligned}
u &= \overline{u} + u' \\
v &= \overline{v} + v' \\
h &= \overline{h} + h':
\end{aligned}
$$

Without going over the details here, a time evaluated integral is completed and we get the following,

$$
\frac{1}{T}\int_0^T (huv)_x \, dt = \overline{h}\,\overline{u}\,\overline{v} + \text{other terms};
$$

where the other terms include Reynolds averaged terms.

We find that the terms in this integral, contribute to something physical in the basin. This is known as eddy diffusion. Eddy diffusion comes about from the time-averaging process via the Reynolds terms and becomes apparent when we have turbulent motion. Averaging in time shows this eddy diffusion happening. This is the reason for why the diffusivity, A, has to become larger. The tenfold increase of

5 Application on an Open Basin: Open North and South Boundaries

In the next section, we examine the case discussed previously but this time the knowledge of mass transport, $\int_0^{L_1} hv_1\,dx \approx HV_1L_1 \approx 15 \times 10^6\,m^3s^{-1}$, taking a depth $H \approx 500\,m$.

5.1.1 Zonal Momentum Equation

Integrating equation (5) and using Scaling Analysis as done previously allows us to calculate estimates for the terms in the zonal momentum equation. We find that the first advection term

$$\iint (hu^2)_x\,dx\,dy = 0.$$

We know this because u does not penetrate through the east and west boundaries.

The second advection term is given by

$$\iint (huv)_y\,dx\,dy.$$

We must be careful with this advection term as when subtracting the orders of magnitudes at both limits, we can get a cancellation. This is the case when taking the in and outflow as completely northwards. However, taking a component of u, where $u = 0.01\ ms^{-1}$, westwards, at the northern boundary and $u = 0.01\ ms^{-1}$, eastwards, at the southern boundary then there is a momentum contribution. This will correspond to the outflow flowing very slightly to the northwest and inflow flowing very slightly to the northeast. Taking these u values gives us a value for this approximation as

$$\iint (huv)_y\,dx\,dy = \int huv\Big|_{y=0}^{y=M}\,dx \approx 3 \times 10^5\ m^4s^{-2}.$$

The Coriolis term can be approximated at mid-basin. This is valid because the value is of a similar magnitude here, compared to the northern and southern boundaries of the basin. We take

$$= \int Ahu_y \Big|_{y=0}^{y=M} dx \quad 1.5 \quad 10^4\, m^4 s^{\;2}.$$

We note that the values for the current along the north and south boundaries are found using continuity, where $\int hu\, dy = 15\, Sv$, giving a value of $u = \;\; 0.03\, ms^{\;1}$ in the northern half of basin. This is equal and opposite in the southern half of the basin. Furthermore, u does not penetrate the east and west boundaries, so the first term vanishes.

The wind stress, as seen earlier, gives no contribution, so

$$\iint \frac{(x)}{} \, dx\, dy = 0.$$

The pressure term depends on the depth of water at the east and west boundaries. The Coriolis term dominates the momentum equation here and we know that the pressure term is the only term that can now balance the equation. We find that

$$\iint \tfrac{1}{2} g^0 (h^2)_x \, dx\, dy \qquad 3 \quad 10^9\, m^4 s^{\;2}.$$

We find that for this open basin, the result is the current is in geostrophic balance. This is where the Coriolis term and pressure term dominate the equation. They act in order to balance each other.

Figure 6 shows how the two forces act upon a fluid parcel, **u**.

Figure 8: Geostrophic Balance.

The two forces act at a 90 angle towards the fluid parcel. The equation that governs geostrophic balance is

$$fv = \;\; \frac{1}{}\frac{\partial p}{\partial x} = \;\; g^0 \frac{\partial h}{\partial x}.$$

Furthermore, by looking at the orders of magnitude, we can find out the height profile at the eastern and western boundaries.

The Coriolis and pressure terms dominate the zonal momentum equation. The pressure term is negative, and we find that

$$\int \tfrac{1}{2} g^0 [h^2]_{x=0}^{x=L} < 0$$

which tells us that $h(x = L) < h(x = 0)$.

This situation of water flowing in at the southern entrance, around the gyre and then leaving at the northern exit gives us geostrophic balance. This fact tells us something about the depth profile in the zonal direction. We have now found that the momentum balances tell us that the layer is deeper where the water enters and leaves the basin compared to where it flows around the gyre near the eastern boundary.

This must mean that when the water flows around the gyre, it is spreading out horizontally rather than vertically. We may expect that this is due to the action of the Coriolis and pressure force balance we have in this situation. It may also be explained by the existence of the wind driven gyre, where the flow near the eastern boundary, the northward transport, is far wider than the western boundary current. This may encourage the entering water to also follow a similar width profile away from the western boundary.

5.1.2 Meridional Momentum Equation

We now move onto the meridional momentum equation. We estimate the terms in equation (6) and get the following

$$\iint (huv)_x \, dx \, dy = \int \left[(huv)\right]_{x=0}^{x=L} dy = 0.$$

The term is zero as u = 0 on the eastern boundary and u = 0 on the western boundary, as we found previously.

For the north-south advection term in the meridional case we notice that the flow entering the basin is the same as the flow leaving the basin. This gives us a cancellation, and this term is zero. So,

$$\iint (hv^2)_y \, dx \, dy = \int \left[hv^2\right]_{y=0}^{y=M} dx = 0.$$

The Coriolis term must be split into its beta approximation, as there is a contribution due to a difference in latitude. We recall that the mass transport in the southern half of the basin is exactly the same as the transport in the upper half of the basin. This means we get the following

$$\iint f hu \, dx \, dy = \iint f_0 hu \, dx \, dy + \iint \quad yhu \, dx \, dy = \iint \quad yhu \, dx \, dy.$$

For the remaining term, we split the basin into half. Upon noting the sign change of u and integrating, we find that

$$\iint yhu\,dx\,dy = \int hu \left(\int_0^{M/2} y\,dy - \int_{M/2}^{M} y\,dy \right) dx$$

$$\frac{1}{2} HU \int \left((M/2)^2 - (M^2 - (M/2)^2) \right) dx = -HU \frac{M^2}{4} L_1.$$

The approximation of u inside the basin is unknown for this situation so w b-ing term, we split th

$$\frac{1}{2}g'(h^2)_y\,dx\,dy \quad 5.3 \quad 10^8\; m^4 s^{-2}.$$

Now we find no contribution from the advection terms in the meridional momentum equation. This time the pressure term dominates the momentum balance along with the Coriolis term, which maintains its strength from the gyre. Once again, like the closed basin this tells us that the layer is deeper at the northern boundary since we have a balance in which its sign is positive.

So both equations give us a geostrophic relationship that relates the pressure force on a fluid parcel with the Coriolis force on the same parcel, of which they act in opposite directions.

5.2 Scenario 2: Southern Inflow and Southern Outflow

Our second case of an open basin is now looking at mass transport entering the gyre at the southern boundary adjacent to the western boundary. This mass transport then flows eastwards eventually leaving the basin at the southern boundary adjacent to the eastern boundary. This situation can be seen in Figure 9 below.

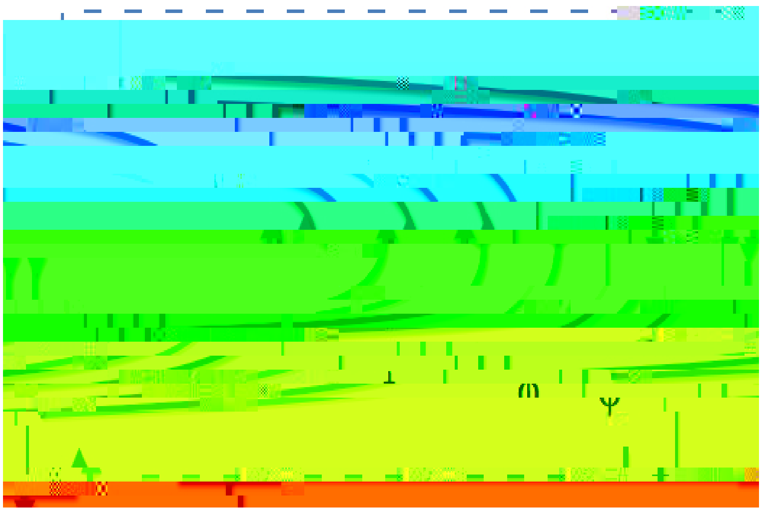


Figure 9: Anticyclonic Gyre Within an Open Basin: Second Scenario.

We use an amount of 15 Sv for the inflow and 15 Sv for the outflow. The length scale of water flowing in and out is taken to be L_2 = 100 km. Using our knowledge of mass transport, we can estimate the velocity of the water flowing in, and this will be the same for the water flowing out.

In the exact manner as done in the first scenario, taking $h = 500$ m, we find that

$$\int hv_2\,dx = 15\,Sv \Rightarrow HV_2L_2 \approx 15 \times 10^6\,m^3s^{-1} \Rightarrow V_2 \approx 0.3\,ms^{-1}.$$

We have the velocity, v_2 for the mass transport entering this will be equal but opposite for the mass transport leaving. We are now a position to estimate the momentum equation for this scenario.

5.2.1 Zonal Momentum Equation

The first advection term is zero due to the boundary conditions at $x = 0$ and $x = L$, i.e.

$$\iint (hu^2)_x\,dx\,dy = 0.$$

The second advection term is also zero. This is still the case if we take the inflow and outflow as not entirely meridional, say, it has a component $u = 0.01\,ms^{-1}$, flowing eastwards. The terms cancel on the southern boundary,

$$\iint (huv)_y\,dx\,dy = \int huv\Big|_{y=0}^{y=M} dx = -\int huv\Big|_{y=0} dx = 0.$$

The Coriolis parameter and the mass transport are of equal magnitude but opposite sign for inflow and outflow, so upon adding both contributions we get

$$\iint fhv\,dx\,dy = 0.$$

The friction term integrated over the basin is also as we found earlier

$$\iint A\nabla\cdot(h\nabla u)\,dx\,dy \approx 5000\,m^4s^{-2}.$$

Similarly to the gyre scenario in the closed basin, we find the pressure term must also be small in order to get a correct momentum balance. It must balance with the total contribution from the other terms. So we find that

$$\iint \tfrac{1}{2}g'(h^2)_x\,dx\,dy \approx 5000\,m^4s^{-2}.$$

Unlike the previous situation there is no acting Coriolis term on this flow and for this momentum equation to hold, the depth at the eastern boundary must be approximately equal to the depth at the western boundary, as found in the closed basin earlier.

5.2.2 Meridional Momentum Equation

Now we look at the meridional momentum equation and find approximations for the the following terms like we have seen previously.

The first advection term is zero due the boundary conditions at $x = 0$ and at $x = L$,

$$\iint (huv)_x \, dx \, dy = 0.$$

The second advection term now contributes since the sign of v is always positive, we find that

$$\iint (hv^2)_y \, dx \, dy = \int \left[hv^2 \right]_{y=0}^{y=M} dx = -\int \left. hv^2 \right|_{y=0} dx \approx -9 \times 10^6 \, m^4 s^{-2}.$$

The Coriolis term is calculated using mass continuity since we do not know what the velocity u is as the water flows eastwards along the southern boundary.

In Section 4.1.2, we saw the method used to approximate a value for u, we use the same method here, taking the same small area and find that

$$HU_2 \tfrac{M}{2} = HV_2 L_2.$$

We now estimate the Coriolis term over the southern half of the basin, where the water is flowing, and we find that

$$\iint fhu \, dx \, dy \approx fHU_2 L \tfrac{M}{2} = fHV_2 L_2 L \approx 7.5 \times 10^9 \, m^4 s^{-2}.$$

Since this mass transport acts in addition to the gyre within the basin, we must add the Coriolis force contribution from the gyre to give us a total contribution of $\approx 8 \times 10^9 \, m^4 s^{-2}$.

The friction terms are first estimated for the inflow and outflow which do not cancel here, the approximation is

$$\iint A \nabla \cdot (h \nabla v) \, dx \, dy \approx 3 \times 10^5 \, m^4 s^{-2}$$

Now adding the friction created by the gyre, we have a total contribution of $\approx 7 \times 10^5 \, m^4 s^{-2}$

The pressure term over the basin in the meridional direction must balance with the Coriolis term so we find that

$$\iint \tfrac{1}{2} g' (h^2)_y \, dx \, dy \approx -7.5 \times 10^9 \, m^4 s^{-2}.$$

We find firstly that the meridional advection term in this equation gives us a contribution worth mentioning but the Coriolis and pressure terms are dominant and again balance the equation giving us a geostrophic balance between the two terms. This concept holds true for this flow into the basin, in this direction.

However, in the zonal equation we did not find a geostrophic balance. This can be explained by the current not having a net Coriolis contribution for the inflow and outflow. In the next scenario, when looking at a current similar to this one but where some transport is deflected northward and around the gyre, we should find the results differ due this deflection of current.

5.3 Scenario 3: Southern Inflow with Southern and Northern Outflow - Realistic Situation of the Antarctic Circumpolar Current

The next scenario we look at considers a realistic example of the South Atlantic Ocean where mass transport enters and leaves as combination of Scenarios 1 and 2; entering at the southwestern boundary and leaving at both the southeastern and northwestern boundaries, see Figure 10 below.

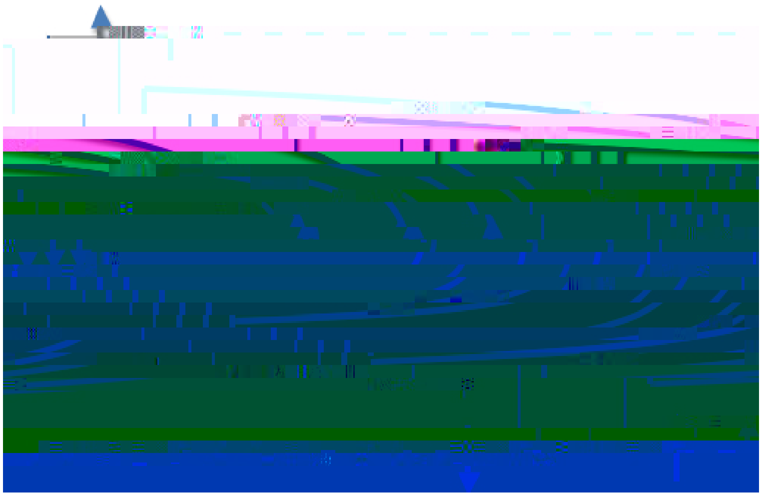


Figure 10: Anticyclonic Gyre Within an Open Basin: Third Scenario.

The example tries to imitate the Malvinas Current and part of the Antarctic Circumpolar Current (ACC). The Malvinas Current is a branch of the ACC that flows northward directly east of South America. The ACC is a current that completes a whole circuit around the Earth, and is an important ocean current in the Southern Ocean. It encircles the Antarctic continent and whilst it does so it enters the southern basins of the Atlantic, Indian and Pacific Oceans. We are concerned with the ACC entering the Southern Atlantic Ocean.

In terms of mass transport, the ACC is a dominant current in the southern hemisphere. It is estimated that the ACC provides approximately a mean transport of 134 13 *Sv* (Whitworth, 1983; Whitworth and Peterson, 1985) through the Drake Passage located below South America. From this approximation, we assume that the inflow in our scenario is approximately 80 *Sv*. We know that much of this easterly flowing current continues through to the Indian Ocean, so we assume that 65 *Sv* leaves at the southern boundary and only 15 *Sv* makes it around the basin and leaves at the northern boundary.

Estimating the velocities of the transport entering and leaving the basin is done in a similar fashion as seen previously.

We find that the inflow has velocity $v_3^1 = 1{:}6\,ms^{\ 1}$, the outflow at the southern boundary has velocity $v_3^2 = \ \ 1{:}3\,ms^{\ 1}$ and the outflow at the northern boundary has velocity $v_3^3 = 0{:}3ms^{\ 1}$. We have assumed that the depth, $h = 500\,m$ at both north and south boundaries and that the length scale of inflow and outflows are all given by $L_3 = 100\,km$.

We now estimate the terms in the momentum equations using the information above.

5.3.1 Zonal Momentum Equation

The first advection term is once again zero due to no normal flow at the boundaries at $x = 0$ and $x = L$. This continues to hold because the east and west boundaries remain closed. Hence

$$! \quad (hu^2)_x\,dx\,dy = 0 \; 3 \qquad :3^{1}$$

$$\int hu_3^3 v_3^3\,dx \qquad \int hu_3^1 v_3^1\,dx + \int hu_3^2 v_3^2\,dx \qquad 3 \quad 10^5\; m^4 s$$

We know the Coriolis effect increases with increasing latitude, however, we must

a Natal Pulse event, where the northern part of the current has intermittent solitary meanders. These propagate downstream and occur at around 6 times per year (Lutjeharms and Roberts, 1988; Lutjeharms et al., 2003). If these meanders reach the retroflection, we expect that a ring shedding event will proceed (van Leeuwen et al., 2000).

In this scenario we will examine the existence of the Agulhas Current in the South Atlantic Ocean and look at the momentum balances. We will for now assume that all of the mass transport that enters the South Atlantic Ocean, retroflects back into the Indian Ocean. So we have an inflow of $65\,Sv$ and also an outflow of $65\,Sv$. These will occur on the eastern boundary of our basin (Figure 11).

We find that the first advection term now gives us a contribution, this is of order of magnitude given by

$$\iint (hu^2)_x \, dx \, dy = 1.7 \times 10^8 \, m^4 s^{-2}.$$

The second advection term is zero in this case. There is no advection occurring on the north or south boundaries, so

$$\iint (huv)_y \, dx \, dy = 0.$$

As we can see from Figure 11, there must be a Coriolis effect on the mass transport that is flowing southwards on the so called meander, where the current retroflects. The Coriolis term in this equation is approximated by the following

$$\iint f h v \, dx \, dy \approx f H V_4 L_4 M_5,$$

where L_4 is the zonal length scale of the current from where it begins to flow in on the eastern boundary, to where it diverts southwards and M_5 is the meridional length scale of the area where the current is acting on. These length scales can be seen in Figure 12. We take $L_4 = 500\,km$ and $M_5 = 400\,km$.

Figure 12: Length scales of Agulhas Current and retroflection area

In order to complete the approximation, we need a value to take for $v_4 \approx V_4$.

Using mass continuity as seen earlier, we have $HU_4M_4 \approx HV_4L_4$.

This now means we can find an approximation to the Coriolis term, by substitution we get

$$! \quad fhv\,dx\,dy \quad fHV_4L_4M_5 \quad fHU_4M_4M_5 = 2{:}6 \quad 10^9\,m^4 s \;\; ^2.$$

Note, the sign is positive due to f being negative and the direction of the u_4 being westwards and v_4 being southwards respectively means that the velocities are also negative.

The friction term for this scenario is

$$! \quad A\,\mathrm{r} \quad (h\,\mathrm{r}\,u)\,dx\,dy \quad 6{:}5 \quad 10^5\,m^4 s \;\; ^2$$

We find again that the balance so far cannot be true unless the pressure term has its contribution. The pressure term should balance with the advection and Coriolis terms which gives us

$$! \quad \tfrac{1}{2}g^0(h^2)_x\,dx\,dy \quad 2{:}4 \quad 10^9\,m^4 s \;\; ^2.$$

The results tell us that for the Agulhas Current and Return Current entering the South Atlantic Ocean, the flow is likely to be in geostrophic balance. However, we have a large contribution also from the zonal advection term in this case. We understand

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5.4.2 Meridional Momentum Equation

Now we we look at the meridional momentum equation. The first advection term is zero. Earlier we saw that we can find a contribution to a term similar to this one

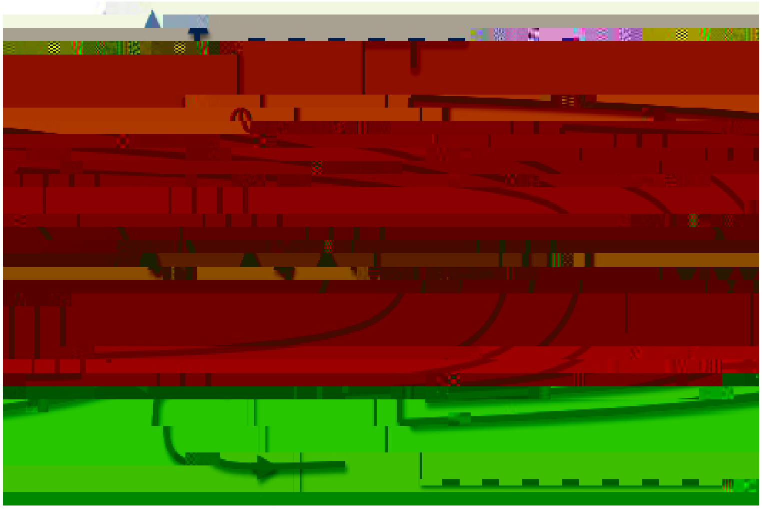


Figure 13: Anticyclonic Gyre Within an Open Basin: Fifth Scenario

The second advection term is zero if the outflow is completely northward. However, it can be non-zero in this scenario. We will assume that a zonal velocity of $u = -0.01\,ms^{-1}$ affects this outflow in order to get a contribution, we find that

$$\iint (huv)_y\,dx\,dy \approx 1.5 \times 10^5\ m^4s^{-2}.$$

We estimate the Coriolis term using the methods used in Scenarios 2 and 4. In scenario 2, we used a value of f at mid-basin and approximated the Coriolis term for the mass transport flowing northwards near the eastern boundary and in Scenario 4, we approximated it in the smaller area where the Agulhas Current was flowing. Combining the two methods we get an approximation to this term given by

$$\iint fhv\,dx\,dy \approx fHV_5L_5(M - M_5) + fHU_5M_4M_5 \approx 8 \times 10^8\ m^4s^{-2}.$$

The friction term in the zonal direction is

$$\iint A\,\nabla\cdot(h\,\nabla u)\,dx\,dy \approx .5 \times 10^8\ m^4s$$

We find here something similar to what we found in the previous scenario. The advection due to the Agulhas Current entering the basin is a term which is included in the momentum balance as well as the pressure and Coriolis terms.

As mentioned earlier, we suggested that the Agulhas Current's estimation may have uncertainties involved with it. Here we have taken a larger value for the inflow than used previously so we expect that the advection term will be larger also due to extra momentum in the basin. A larger estimate for the inflow has been used to account for mass transport that flows out at the northern end of the basin and still retain a reasonable value for the Agulhas Return Current.

So here we find a balance between three terms, which include an advection term, the pressure term and the Coriolis term. This balance provides us with a different relationship to what we found when we had geostrophic balance. This balance has no theoretical explanation behind it and the occurrence of the extra advection term implies we do not have a geostrophic flow.

Here we can also expect that the balance can be between the Coriolis and advection terms, this is a possible momentum balance. Although the local balance within the basin can be mainly geostrophic, the large-scale integrated balance can be different.

The extra momentum related to the advection here could be related to the eddy formation and Agulhas rings. The existence of this term has appeared in both the Agulhas Leakage scenarios in the zonal direction. The Agulhas rings may be the reason for why this extra momentum is appearing in the advection term.

5.5.2 Meridional Momentum Equation

Next we examine the meridional momentum equation. Firstly, the first advection term in this equation depends on the direction of the inflow and outflow of the Agulhas Current. Previously in the zonal equation, we stated that we would use no component of a velocity v affecting the this current into and out of the basin. If so, then this term is zero. If there was a southward component of, say $v = -0.01\, ms^{-1}$, then the contribution is

$$\iint (huv)_x \, dx\, dy = 1.5 \times 10^5 \; m^4 s^{-2}.$$

The second term is advection on the north and south boundaries. We have advection occurring on the northern boundary and this is given by

$$\iint (hv^2)_y \, dx\, dy = 4.5 \times 10^6 \; m^4 s^{-2}.$$

The Coriolis term in this equation is split into two different approximations. One uses the same method as seen in the previous scenario and the other approximates the Coriolis term on the mass transport flowing westwards in the northern half of the basin, similar to the approximation is Scenario 3.

Firstly, the contribution from the Agulhas Current inflow and Agulhas Return Current outflow, is given by

$$\iint fhu\,dx\,dy = \iint (f_0 + \beta y)hu\,dx\,dy$$

$$= \iint_0^{M_5/2} f_0hu\,dx\,dy + \int \tfrac{y^2}{2}hu\big|_{y=0}^{y=M_5/2}\,dx + \iint_{M_5/2}^{M_5} f_0hu\,dx\,dy + \int \tfrac{y^2}{2}hu\big|_{y=M_5/2}^{y=M_5}\,dx\; m^4s^{-2}$$

$$\approx 4\times 10^9 + 1.3\times 10^8 - 3.25\times 10^9 - 4.8\times 10^8 = 4\times 10^8\, m^4s^{-2}.$$

Secondly, is the approximation for the westward flow in the northern half of basin. Using mass continuity, we know that $15Sv$ leaves the basin at the northern boundary, so the westward flow of the transport has velocity $u_5 = -0.03\,ms^{-1}$ over the distance of half the basin, i.e. $1000\,km$. We find the contribution given by this flow is

$$\iint (f_0 + \beta y)hu\,dx\,dy \approx f_0HU_5L\tfrac{M}{2} + \tfrac{3\beta(M)^2}{8}HU_5L \approx 5.3\times 10^9\, m^4s^{-2}.$$

The total contribution of the Agulhas Current and westward flow in the basin is $5.7\times 10^9\, m^4s^{-2}$ We must add the contribution from the gyre to this and we have the full approximation of the Coriolis term which is given by

$$\iint fhu\,dx\,dy \approx 5.2\times 10^9\, m^4s^{-2}.$$

The friction term for this scenario is

$$\iint A\nabla\cdot(h\nabla v)\,dx\,dy = 2.5\times 10^3\, m^4s^{-2}.$$

The pressure term here must act to balance this equation and is given by

$$\iint \tfrac{1}{2}g'(h^2)_y\,dx\,dy \approx -5.2\times 10^9\, m^4s^{-2}.$$

In the zonal momentum equation, we found a balance that involved an advection term along with the Coriolis and pressure term. However, in this direction, we are back to having the latter two terms being dominant.

We find that the pressure term in this direction is negative for the momentum equation to balance. Physically this means that the depth of water in the active layer is now shallower at the northern boundary than at the southern boundary.

In terms of a geostrophic flow, we know that the Coriolis term acts towards the left of the flow and the pressure gradient is towards the right. Here, we discover the opposite occurring and find an interesting result.

6 The Agulhas Influence on the Overturning Circulation. What is actually going on?

From examining the different scenarios of mass transport into and out of the basin in Section 5, we are now in a position to put them all together and configure the realistic situation. We will again use the momentum equations to find out how the Agulhas leakage is influencing the overturning circulation. We know where the water enters and leaves the basin, we also have estimates for the amount of water flowing in and out, but we are unsure of what the mass transport is actually doing within the basin. We aim to look at what the momentum balances can tell us about the South Atlantic Ocean.

The Agulhas leakage is where large oceanic rings are shed from the retroflecting Agulhas Current. It is an interocean exchange between the Atlantic and Indian Oceans (W Weijer, 2002). The Agulhas leakage has an important influence on the Overturning Circulation, providing heat and salt into the Atlantic Ocean. A change in the Agulhas leakage could lead to major changes on the Overturning Circulation. Increasing freshwater fluxes can cause the Ocean's Conveyor to completely shut-down (W. S. Broecker, 1991) and may only be overcome by increased salinity in the Atlantic Ocean. These changes can cause abrupt climate changes and it is important to realise the ocean plays a dominant role in this along with the atmosphere.

There are two possible situations in which we think the Agulhas leakage could have an impact on the circulation. The first is where part of the Agulhas current that retroflects back into the Indian Ocean, flows northwards around the gyre, then westwards towards the western boundary and then northwards again out the basin. In addition to this is, we have the ACC that enters at the southwest and flows eastwards out into Indian Ocean. We assume that the ACC does not contribute to the transport leaving the basin at the northern boundary. A diagram of this situation can be see in Figure 14.

The second situation is similar to the above except that the Agulhas Current that retroflects provides no diverted mass transport. In other words, all the mass transport that enters on the eastern boundary, leaves on the same boundary. The difference is that the ACC provides the mass transport that leaves the basin at the northern boundary, however, most of this current also leaves the basin where the Agulhas Return Current leaves. This situation is depicted in Figure 15.

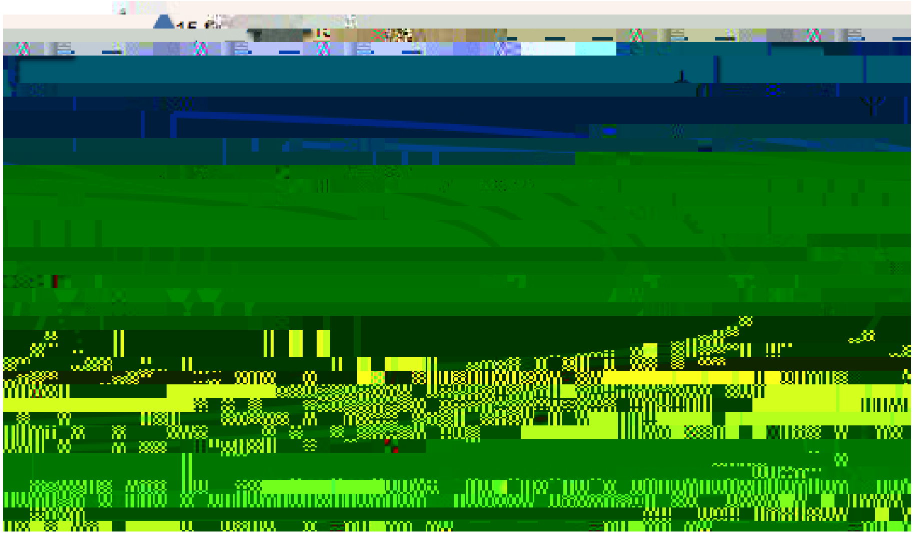


Figure 14: Configuration suggesting the northern outflow is from the Agulhas leakage.

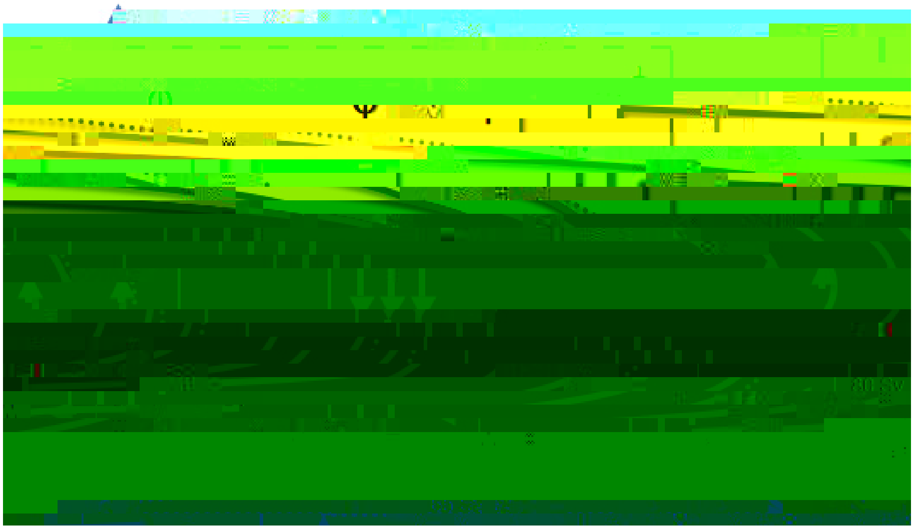


Figure 15: Configuration suggesting the northern outflow is from the ACC.

6.1 Momentum Balances for the Agulhas Leakage Providing the Outflow at the Northern Boundary

As seen in x**5**, we return to the momentum equations and try to figure out the momentum balances for the situation where the Agulhas leakage provides the outflow at the northern boundary of our basin.

We begin with the advection terms in the zonal momentum equation. The advection we have here is on the eastern boundary and from the Agulhas Current, Return Current and the easterly flowing ACC. We find that

$$\int (hu^2)_x \, dx \, dy \quad 5.5 \quad 10^8 \, m^4 s^{\ 2}.$$

The Coriolis term in this equation must be split into three different approximations, all of which we have seen previously. These take into account the Coriolis force acting on the water flowing eastwards, westwards and the water that is apart of the Agulhas Current and retroflection.

The contribution from the Agulhas Current inflow and Agulhas Return Current outflow is the same as that calculated in the previous scenario. This is given by

$$\int f hu\,dx\,dy = \int (f_0 + \quad y)hu\,dx\,dy$$

$$= \int_0^{M_5=2} f_0 hu\,dx\,dy + \int \frac{y^2}{2} hu\Big|_{y=0}^{y=M_5=2} dx + \int_{M_5=2}^{M_5} f_0 hu\,dx\,dy + \int \quad y^2$$

We begin with the advection terms, they are the advection on the boundaries and since this is the same as the previous case, they are no different. They are

$$\iint (hu^2)_x \, dx\, dy \approx 5.5 \times 10^8 \; m^4 s^{-2}$$

and

$$\iint (huv)_y \, dx\, dy = 0.$$

The Coriolis term however, is different, it depends on the transport flowing northward from the ACC and the southward flowing Agulhas Current. We have

$$\iint fhv \, dx\, dy \approx fHV_{north}L_5(M - M_5) + fHU_{80sv}M_4M_5 \approx 2 \times 10^8 \; m^4 s^{-2}.$$

The friction term is given by

$$\iint A \nabla \cdot (h \nabla u) \, dx\, dy \approx 4.8 \times 10^6 m^4 s^{-2}.$$

In order to balance the equation, the pressure term now turns out to be

$$\iint \tfrac{1}{2} g'(h^2)_x \, dx\, dy \approx -3.5 \times 10^8 \; m^4 s^{-2}.$$

We also find the approximations for the meridional momentum equation. The advection terms are the same as the previous situation where

$$\iint (huv)_x \, dx\, dy = 0$$

and

$$\iint (hv^2)_y \, dx\, dy = 1.2 \times 10^8 \; m^4 s^{-2}.$$

The Coriolis term is split into three cases, firstly looking at the transport of 15 Sv that flows eastwards then westwards, then for the 65 Sv eastward transport that flows out with the Agulhas Return Current and the third is for the Agulhas Current.

We have for the 15 Sv transport

$$\iint fhu \, dx\, dy = \iint \beta yhu \, dx\, dy \approx \frac{M^2}{4}HUL_4 = -3 \times 10^{88} \; (8$$

$$\iint f hu\,dx\,dy \approx fHULM_4 \approx 3.3 \times 10^{10}\, m^4 s^{-2}.$$

Finally, for the Agulhas Current we have

$$\iint f hu\,dx\,dy = \iint yhu\,dx\,dy = \int \frac{y^2}{2} hu|_{y=0}^{y=M_5/2}\,dx + \int \frac{y^2}{2} hu|_{y=M_5/2}^{y=M_5}\,dx$$

$$\approx 1.6 \times 10^8 \approx 4.8 \times 10^8\, m^4 s^{-2},$$

since we know that the amount flowing westwards is equal to the amount flowing eastwards.

The total contribution of the is $3.3 \times 10^{10}\, m^4 s^{-2}$: We must add the contribution from the gyre to this and we have the full approximation of the Coriolis term which is

$$\iint f hu\,dx\,dy \approx 3.4 \times 10^{10}\, m^4 s^{-2}.$$

6.2 Does the Agulhas Current Provide the Outflow at the Northern Boundary?

In Section 6.1 we found estimates of the terms in the momentum equations with two different scenarios put out. We are looking to obtain an answer around what is providing an outflow at the northern boundary of the South Atlantic Ocean. We have used scenarios that suggest it is either the Agulhas Current or the ACC.

From the approximations evaluated previously, we find that the momentum balances are highly similar in both cases. The advection terms, pressure terms and friction terms do not change in the two cases and although the two situations are different, the integrated momentum equations do not fully distinguish between the two. The flows at the boundaries and within smaller areas, for example the area in which the Agulhas Current flows into, do not provide major differences to be noticed in the the momentum equations. However, we are concerned with the momentum balances; momentum has to be conserved and therefore, we must look carefully at this.

Starting in the meridional momentum equation, the two cases are almost exactly the same in regards to every term. A difference we notice is the Coriolis term that differs by $2 \times 10^{9}\,m^4 s^{-2}$. In comparison to the meridional advection term, this is in the zonal direction.

The result we have is that the Agulhas leakage situation has a momentum balance that implies that the height profile at the eastern boundary is greater than the height profile at the western boundary. However, the ACC gives us the opposite result, where the depth is greater at the western boundary than the eastern boundary.

With the Agulhas leakage configuration, the depth profile shows a positive gradient (from east to west). The transport that travels away from the Agulhas leakage may have an effect on this. We suggest that here most action is taking place, in comparison to the western boundary causing a deeper layer at the this boundary. The momentum due to the mass transport at this could be having an impact here.

Whereas the ACC provides the opposite zonal profile. This may suggest that the transport diverted from this current does not flow as close to the eastern boundary as expected. It may either spread out and become a wider current whilst it travels northwards, or it flows northward closer to the centre of the basin. Either idea may be the reason for why the western boundary is deeper than the eastern boundary.

The above gives us some important results that have been found from the large-scale momentum balances. We must note here that the gyre enclosed in the basin has no effect on the zonal Coriolis term and advection terms coming from the momentum equation. We know that the pressure term has to balance with these terms.

Aside from the momentum balance, we must be aware of any discrepancies in the estimated terms. Oceanographic studies have already provided us with a good balances in each of the scenarios used so far.

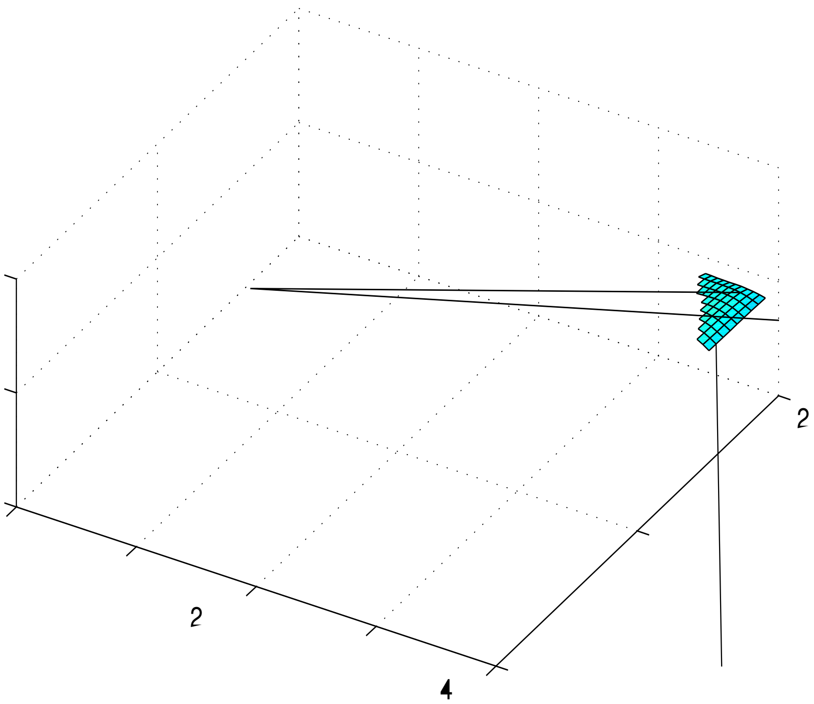
Returning to the momentum balances, we find that it is difficult to say we have one clear answer. We cannot confirm where the diversion comes, either from the Agulhas leakage or the ACC. This leads us to a situation where measurements are be taken to confirm which configuration we have in the South Atlantic. We find important results from the momentum balances here but leads us to a study that entails depth measurements in which future studies should incorporate. However, with that option unavailable to us, we try to address this problem using the vorticity balances.

7 Vorticity Balances on the South Atlantic Ocean

7.1 Numerical Model Outputs

We now focus on the numerical model to provide us with visual and numerically calculated terms in the vorticity equation. We begin to look at numerically generated plots to try to aid our understanding of the vorticity balance when the Agulhas leakage is introduced into the basin.

Figures 16 and 17 show the numerical model's output of the South Atlantic Ocean, separated by an island on the eastern boundary that can be amended to provide us with closed and open basins respectively.



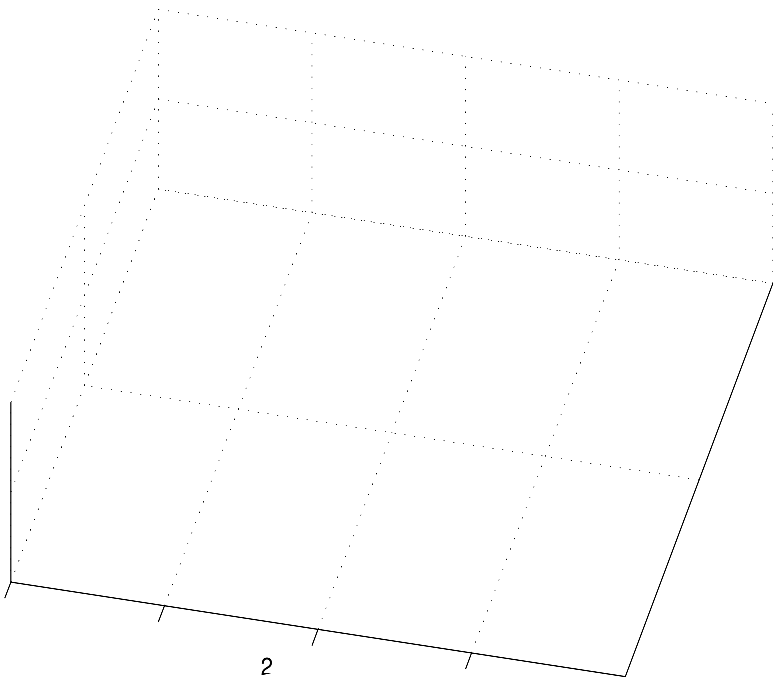


Figure 17: Open basin: Configuration of wind-driven gyres of the South Atlantic and Indian Ocean basins with an island, partly open, allowing a connection between the two. Interface (e) is given by e = h - 500 m.

the South Atlantic in these model outputs here, this is given by $0{:}2\,Pa$. This value is chosen here to clearly show vorticity within the open basin. The plots have been taken at a model run-time of 1000 days. We notice that in the South Atlantic the action is occurring mostly at the southwest of the basin. We expect that due to the western boundary current, large amounts of friction is produced. Vortices that we can see are produced by instabilities of the western boundary current and due to the separation of this current from the coast in the south of the domain. Friction is due to large gradients in the velocity fields and we see this when the vortices are formed.

We also recognise that this is there are not much taking place in the rest of this basin. We know the currents are smaller elsewhere and is demonstrated in the numerical model. We must pay particular attention to the eastern boundary where the island is closed at the moment as this is where we expect to find an interruption from rings shed here.

Figure 17 shows an open basin, where the island does not extend all the way down to the southern boundary. This produces an output showing the influence of the Agulhas leakage in the ocean basin. Previously, near the eastern coast of the closed basin, there was not much happening. However, now we notice the shedding of rings into the basin. The friction at the western boundary is still there and acts over a larger area than previously.

The movement of the rings into the basin occur separately suggesting they are shed at different times. After 3 years, we notice at least 3 rings have been shed into the basin. After 3000 days we will have more than 3 rings produced. Their average speed will be approximately R_d^2 which is a few cms ^{1}. The interaction of the rings in the South Atlantic Ocean is now apparent.

Up until now we have looked thoroughly at the momentum equations, examining them on the closed basin and different situations of an open basin. This lead us to a realistic situation involving the Agulhas leakage and ACC. We have so far found many scenarios that were governed by geostrophic balance and others that were more complicated. These balances included advection terms.

Understanding the Agulhas leakage influence on the South Atlantic by looking at the vorticity equation is to be studied. We previously looked at vorticity in the closed basin and found from this that the diffusivity coefficient should be larger to provide a vorticity balance. However, we noticed it was difficult to approximate the vorticity stretching term since it cancelled due to the geostrophic relationship.

We now wish to look at the vorticity balance of Agulhas leakage but this time by using our numerical model. The model generates two wind-driven gyres being the South Atlantic and Indian Oceans, separated by an island which can be modified to give us a closed basin or an open basin in which the Agulhas leakage can enter the basin; a model of the real world phenomenon. We note that no numerical runs with open boundaries, around the basin itself have been done because these lead to ill-posed numerical problems that are hard to solve.

Furthermore, by numerically calculating the terms in the vorticity equation, we are able to overcome the difficulty in analytically calculating the stretching term and find estimates for all the terms over the basin and see where they are acting within the basin. In regards to estimating the terms, we average them over a given time, which does not start at the initial run of the model as the model has to reach a statistical equilibrium first before this can take place. We know large Agulhas rings are shed from the Agulhas Retroflection and we expect to see what the vorticity balance in light of the numerical model can tell us.

7.2 Vorticity Within the Basin

7.2.1 Closed Basin Integrated Terms

Using our numerical model we now look at the basin integrated terms in the vorticity equation. We begin with the closed basin, where the island fully encloses the South Atlantic. The terms outputted are the time-averaged terms, for a run of a 1000 days and averaged from 300 days. A run of this length suffices for the closed basin scenario since vorticity from the Agulhas leakage is not going to be apparent here.

The first term is the time dependent term given by

$$(h\ \ )_t = \ \ 3\,ms\ ^{2}.$$

The zonal advection term that we are interested in is

$$(hu\ \ )_x = \ \ 19\,ms\ ^{2}.$$

The stretching term that we stated earlier is difficult to approximate is

$$h(f + \ \ )(u_x + v_y) = \ \ 27\,ms\ ^{2}.$$

The Beta term is

$$hv = 32\,ms\ ^{2}.$$

The wind-stress term is

$$h\ \frac{(x)}{h}\ _y = \ \ 393\,ms\ ^{2}.$$

Finally the friction term is

$$A\ \ \ _{xx} + \ \ _{yy}\ = 412\,ms\ ^{2}.$$

We find that the time dependent term is small as expected; we are running the model to a statistical equilibrium and this term is showing this is true. The zonal advection term within the basin is small also as expect in the closed basin case. We notice that the stretching and Beta term are small and similar in magnitude but have opposite signs; this confirms that they do tend to cancel with each another. We find the dominating terms are the wind-stress and friction terms which act to balance each other and represent the vorticity balance in the closed basin.

Importantly, the vorticity balance we find here in the closed basin from the numerical model agrees with the analytical calculations found earlier for the vorticity equation in Section 4.4. We found that the wind-stress was dominant and the friction term was the only term able to balance it. Here, the numerically modelled integrated terms give us a confirmation of that result.

The result we are interested in, is the comparison of the vorticity balance with the momentum balance. We found that the pressure gradient and Coriolis terms were dominant in the momentum equation. However, now the vorticity balance is seen to be from the wind input and the friction. This shows us how the different sets of equations emphasise different aspects of the physics for our basin, yet the equations originate from the same basis.

7.2.2 Open Basin Integrated Terms

We now look at the basin where the island is open, so it allows the Agulhas leakage to enter the basin. This time, for the ocean to reach a statistical equilibrium we have increased the run-time to 3000 days and taken an average from 1500 days for the time integrated terms.

The time dependent term is

to the meridional pressure gradient. This is related to the extra mass and vorticity input due to the Agulhas rings, but how exactly is unclear.

The time dependent term is also small and this should be the case as mentioned with the closed basin case. We expect that in both closed and open basins that the Beta term is larger due to the western boundary current however, this seems to be encapsulated by the stretching term.

Again friction and wind-stress are also dominant showing a different side to the ocean physics here in comparison to the geostrophy that we find in the momentum balances. With the momentum balances for the Agulhas leakage we found a large advection term in the momentum balance also. The term shows a reasonable contribution but not as much as we would expect to see. The vorticity input by the boundary is still relatively small in comparison to the real world where we find that the Agulhas rings are as large as the vorticity input by wind. A higher model resolution may capture this but is expensive to implement.

Rather then observed inaccuracies in the ocean magnitudes, we find here that the model may not observe the detail that we might expect. Again, the balance it provides us gives us a good insight to our physical understanding but this can always be improved, especially computationally. Increasing the horizontal resolution requires more computational power and this is a limiting factor here. Nonetheless, the model is adequate and does provide enough detail and explains the vorticity balance we have.

7.2.3 Vorticity Fields on the Open Basin

The vorticity equation has different contributions and these individual contributions can be seen as 2-dimensional fields from the model output. We continue to use the 3000 day run and look at the images at this moment in time. These fields can be seen in Figure 18.

Figure 18: Fields of the terms in the vorticity equation.

The vorticity contributions seen as images in Figure 18 show us the interior features of the basin. Here the model parameters remain the same, we note also that the wind-stress forcing over the Indian Ocean is given by $0{:}1\,Pa$.

We find that the two advection terms show wave-like features, we see the troughs and crests visible in both situations. However, we find that there is a balance

stretching term found has a significant impact on the thermocline motions and hence an impact on the Thermohaline Circulation.

The vorticity impacts of the Agulhas rings are seen in the numerical plot of the basin as well as within the vorticity fields. The fields in Figure 18 show that at the eastern boundary the eddy terms are visible in all the terms.

The momentum balances give us an insight to there being an additional advection term within the basin when the Agulhas leakage is present and with our numerical model we notice that the extra vorticity has an impact on the vorticity balance. This vorticity balance tells us that there is an impact from the Agulhas leakage on the large-scale ocean circulation.

8 Impacts of the Agulhas Leakage

The vorticity balances on the open basin show that friction, wind-stress and stretching terms are dominant. The friction and stretching terms are large and we suggest that rings are induced by the significant amount of anticyclonic vorticity within the basin.

The vorticity input from the Agulhas leakage was seen in all the fields generated by the numerical model. They also gave us an insight to the local balances between the terms. We notice that extra mass and vorticity is entering the basin via rings and this is what affects the vorticity balance. Vigorous motion is felt in the basin due to the ring shedding and its major impact is on the stretching term. This vortex stretching has an impact on the thermocline motions which will adjust thermohaline circulations that are of most interest.

From the momentum balances we had obtained an important result from comparing two scenarios sought from the real world, where either the ACC or the Agulhas leakage provides a northern flow out the basin. It remains to be shows by measurement which momentum balance is actually occurring. However, we find the combined vorticity and momentum balances from the Agulhas leakage gives us a good inclination that this is the likely real world situation occurring and informs that it has a major influence on the Thermohaline Circulation.

We now understand that the large contributions of stretching and friction within the South Atlantic are due to Agulhas rings shed in the basin. Each ring shed may

9 Summary

The Agulhas area south of Africa is where the Indian Ocean meets with the South Atlantic Ocean. The Agulhas Current flows into the South Atlantic then retroflects back into the Indian Ocean carrying most of its mass transport back with it. Where this current enters and its retroflection takes place is known as the Agulhas leakage. We are concerned with the influence this has on the Overturning Circulation. Throughout, we have used a two-layer reduced gravity model and integrated the given momentum and vorticity equations on the South Atlantic Ocean. Each variable used has its own approximated order of magnitude for a large-scale ocean circulation in the South Atlantic.

We began with a closed basin, having no mass transport entering or leaving. The momentum balances ultimately provided us with information about the depth of the active layer. We find that the depth is approximately equal at the eastern and western boundaries of the basin but also, it is deeper at the northern boundary in comparison to the southern boundary. Furthermore, we found that the momentum equation in the meridional direction told us that the gyre is governed by a balance dominated by the pressure gradient and Coriolis terms. The use of a numerical model confirmed these depth related results.

The next step was to apply the integrated vorticity equation on the closed basin; found by taking the curl of the momentum equations. Vorticity is used to give us a measure the spin within the basin. We learned that the diffusivity parameter had to be an order of magnitude larger for this equation to balance correctly. A larger diffusivity takes into account eddy diffusion associated with turbulent fluids. It also suggests that there is a time dependence. This was considered in our numerical model; it allowed time-averaging of the vorticity terms after the model reaches a statistical equilibrium.

Furthermore, in terms of the diffusivity we had to recheck the momentum balance. The inclusion of a larger diffusivity parameter within the friction term still gave us the same momentum balance between the Coriolis and pressure term. Most importantly, we discovered that this coefficient can vary in the South Atlantic Ocean. We held the same diffusivity parameter throughout for the analytical studies of the momentum balances. However, the time-averaged terms in the numerical model had a larger effective value for the eddy diffusivity.

Systematically, different configurations were then set up aiming to build up to the real world scenario. This meant that the basin had to be adjusted to having open boundaries. The real world configuration that we were to achieve includes the Agulhas leakage and Antarctic Circumpolar Current (ACC) flowing into the South

Atlantic. From looking at these currents flowing in, the aim was to find which one of the two currents could provide a momentum balance if only one of them were to provide the outflow we have at the northern boundary of the basin.

Coriolis force. This meant that the momentum balance was different to the previous scenarios and we began to find more interesting results.

Having looked at the different scenarios of mass transport in and out of the basin, we were in the position to look at the full problem. The problem put forward was that either the ACC or the Agulhas leakage provided the outflow at the northern boundary of the basin.

The main outcome of the problem was related to the depths at the east and west boundaries of the basin. The ACC problem gave us an insight to the western boundary being deeper than the eastern boundary and the opposite was found for the Agulhas leakage.

The previous scenarios had shown balances that were related to geostrophy whilst some were not; these had an involvement from an advection term. The two scenarios here both had a momentum balance in the zonal direction that involved an advection term as well as a Coriolis term. In order to balance the equation fully, the pressure term was derived from these two terms. Positive for the Agulhas leakage and negative for the ACC; this is how we were able to describe the depth at the two boundaries. Both results proved interesting and without further study of the basin itself, we remain unable to comment on what situation exists in the South Atlantic.

In light of this, we turned to vorticity to obtain further information about the Agulhas leakage and what its impacts are on the South Atlantic. We returned to the numerical model to provide us with the a simulation of the real world Agulhas leakage and an idea of the typical vorticity balances.

The numerical model was able to calculate time-averaged terms in the vorticity equation. The closed basin model gave rise to a balance between the wind-stress and friction terms. We already seen in the analytically integrated vorticity equation that this was the case. However, we found an important result from the open basin. We noticed that the vorticity balance was dominated by friction, wind-stress and stretching terms. This vorticity balance gives us an idea of the influence the Agulhas leakage has on the South Atlantic Ocean and also the Overturning Circulation.

The vorticity fields were able to show the local balances of the open island case. However, the balance seen was the stretching and Beta term balancing with the friction term. We know from the integrated terms that the balance included wind-stress; the fields did not show this because the term was small and positive but once integrated over the whole basin it became large and dominant. Numerical outputs of the open island show vorticity being input into the basin from the Agulhas leakage and we notice that these have an influence on the large-scale balances.

10 Conclusion

The influence the Agulhas leakage has on the South Atlantic is determined by the salt and heat input via these Agulhas rings that spawn in this basin. The momentum balances suggest that these rings are the reason for mass transport leaving north of this basin, given a certain amount of uncertainty not being too large. The momentum balance show signs of local geostrophy but also a contribution from zonal advection into the basin, from which we can infer the depth at the eastern boundary is deeper than the western boundary. Further study is encouraged to support the claim that the Agulhas leakage provides a northern outflow and is also to be done with regards to the active layer depth.

The numerical model shows Agulhas rings moving into the basin and that they have a significant impact on the vorticity balance. We understand from this that these rings must have an impact on the Overturning Circulation. The idea is focused on the heat and salt input into the basin by the rings which transport mass from the Indian Ocean. In the time-integrated vorticity balance, we found the appearance of a stretching term which suggests a large impact on thermocline motions in turn having an impact on the Thermohaline Circulation.

We understand the concept behind the Overturning Circulation; based on heat and salinity fluxes around the worlds oceans. We imagine the situation where the Agulhas rings do not spawn in the Atlantic Ocean. The salinity and heat input would be lowered. We have mentioned a shutdown of the oceans conveyorbelt can be caused by an increased amount of freshwater fluxes into the oceans. A lack of salinity is equally interpreted as an increased amount of freshwater. We know the momentum and vorticity balances are conserved by the Agulhas leakage into the South Atlantic Ocean. It must play an important role in maintaining the ocean's Overturning Circulation.

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