On Appro on n Meshless Me hods

The

Notation for Particle Methods

tches

[illegible]

$$\| \cdot \|_{N} \|_X \quad \inf_{v \in V_N} \| \cdot \|_X$$

[illegible]

1.2 The notion of optimality

[illegible]

$$_n \; - \; \inf_{E_n \subset X, \; \dim E_n \le n} \; \sup_{u \in Y} \; \inf_{v \in E_n} \| \cdot \|_X$$

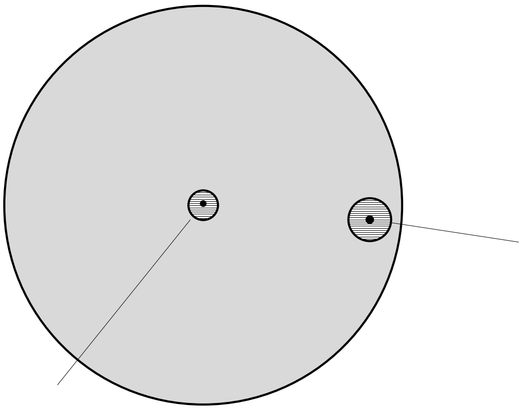
[illegible]

[illegible]

Remark 1.4. [illegible] ■

2 Polynomial Reproducing Systems

[illegible]



Proof of Theorem 2.6. [illegible]

$$\sum_{j=1}^{N} [illegible]$$

[illegible]

$$\sum_{i=1}^{N} [illegible]$$

[illegible]

'd (u583/R48 6. 0 Td +.65974 -1.

pr ... $\mathbb{R}^d$ A ... A

Corollary *Let* $\mathbb{R}^d$ *be Lipschitz dom in. Assume t t the s* e_i *of Theorem 2.6 tisfy dditio y n o e p condition, i.e., for some* $2\ \mathbb{N}$ *e e*

$$p\ r\ f\ 2\ \mathbb{N}\ j\ 2\ e_i\ g$$
$$x\ d$$

Then there exists in r p N 1 $)$ $!$ N *such t t for ch* $2\ \mathbb{N}_0$ *there exists ith*

$$k \quad N\ k_{H^s(\ )} \quad {}^{\min\{p\ 1;k\}-s} k\ k_{H^k(\ )} \quad -$$

Example 2.9. L … 2 … 2 ℕ

[illegible]

[illegible]

[illegible] A [illegible]

[illegible]

[illegible]

A ij n ij A $\mathbb{R}$ p

vthe

$G \in \mathbb{R}^{Q \times Q}$ ... $G_{kl} = \sum_{i=1}^{N}$ ... $a \in \mathbb{R}^{Q}$

$$a \cdot Ga = \sum_{i=1}^{N} \ldots \sum_{k=1}^{Q} a_k \ldots$$

... G ... $a \in \mathbb{R}^{Q}$ ... $a \neq 0$ ... $a \cdot Ga =$ ... $\sum_{k=1}^{Q} a_k$ ...

Exercise 2.14. ...

Exercise 2.15. ...

$$\sum_{i=1}^{N} \ldots$$

Remark 2.16. ...

A

window function

$\mathbb{R}_0 \to \mathbb{R}_0$

tensor product

$\prod_{j=1}^{d}$

Remark 2.17.

$\|\cdot\|$ $\mathbb{R}^d$

$\|\cdot\|_{l^\infty}$ ∎

Regularity of the shape functions

Assumption 2.18.

$\in \mathbb{R}^d$

Remark 2.19.

$\mathbb{R}^d$ ∎

A

Theorem 2 20 *Let* [illegible] *tisfy* [illegible] *cone condition* [illegible] *ith* [illegible] *ng e* [illegible] *nd* [illegible] *dius* [illegible]. *Let* 2 [illegible]), - N − f i j − [illegible] g $\mathbb{R}^d$ [illegible] *nd* f i j − [illegible] g $\mathbb{R}$ [illegible]. *Set*

[illegible] + − f i g − [illegible]

[illegible] *nd* [illegible] *ssume the co* [illegible] *ering condition*

[illegible]

fe j — g

$$\mathbf{j} \quad \mathbf{G}^{-1} \quad) \mathbf{j} \quad -| \; c$$

A

8 2

k j k comp $\frac{1}{}$

3. step:

2. step:

3. step:

$$k \quad -k \quad \frac{\quad}{k_j \quad k} \quad \frac{k_j - k}{k_j \quad k}$$

$$k \quad -k \quad \frac{k_j - k}{k_j \quad k}$$

$$k \quad -k \quad \frac{k_j \quad k}{k_j - k}$$

2 $\frac{1}{\ }$ k −k

4. step: j j j j

2 Bibliographical Remarks

e) —) m m−d) — m−d

■

Example 3.4. A $d';k$ $\in \mathbb{N}_0$ A

		$\mathbb{R}^d$
1;0) —)	0	—
1;1) —))		—
1;) —))		—
;0) —)	0	
;1) —))		
;) —))		

1;0 ;0 k;d' A k;d ■

1 Analysis of a class of RBFs

k k)

Assumption 3.5. k k)

$$ {}^{-1} \quad \|k\|)^{-} \quad) \quad \|k\|)^{-} \quad \forall \in \mathbb{R}^d $$

A

Exercise 3.6. 1;1 A — ■

2 ^( ) = $\frac{1}{(2\ )^d}\int_{\mathbb{R}^d}$ ( )$e^{-ix\cdot}$ d denotes the Fourier transform ^ of a function . The inversion formula takes the form ( ) = $\int_{\mathbb{R}^d}$ ^( )$e^{ix\cdot}$ d .

A

$h \; i$

Z

$$h \; i - d -) -) \quad - f \; jk \; k - h \; i \quad 1 g$$

Proposition *Let … tisfy Assumption 3.5. Then*

1. $\mathbb{R}^d$).
2. $- \; \mathbb{R}^d$) *ith equi… ent norms.*
3. 2 .
4. *is positi…e definite.*

Proof. $\mathbb{R}^d$).

Theorem *Let Assumption 3.5 be … id. Then for distinct points* $N -$ $f \; i \; j -$ g *nd* 2 *the s… ttered interp… tion prob em:*

Find $2 \; N - \; f \; k \; i k) \; j - \; g$
such th… t $i) - \; i) -$

h… s… unique so ution, … hich … tisfies

$$h \quad i - 8 \; 2 \; N$$

… nd

$$k \quad k - \; v \; v_N \; k \quad k$$

Proof. 7 $k \; k$)

$k - \; k \; k k) \; k \; 2 \; N \; b_k) -) \; ix_k$

Z

$$h \quad k i - d - - C \quad ix_k - k) \quad k) -$$

$k \; k$ L

Corollary (stability of scattered data interpolation) *Let* $\mathbb{R}^d$ *be … Lipschitz dom… in (or* $- \mathbb{R}^d$*). Let* $N - f \; i \; j - \; g$ *… nd suppose Assumption 3.5. Then for …* 2)

$$k \quad k_H$$

Proof. [illegible] $\mathbb{R}^d$ [illegible] N [illegible] $\mathbb{R}^d$ [illegible]

$$\| \cdot \|_{H^{k}(\cdot)} \ldots$$

[illegible]

Corollary 10 *Let Assumption 3.5 be satisfied and let* $\Omega \subset \mathbb{R}^d$ *be a Lipschitz domain. Define the* [illegible]

$$\ldots_{i=1,\ldots,N} \| \cdot \| \qquad (5\ldots)$$

Then there exists [illegible] *such that for* [illegible] *there holds*

$$\| \cdot \|_{H^{s}(\cdot)} \ldots^{-s} \| \cdot \|_{H(\cdot)}$$

Proof. [illegible]

1. step: [illegible] N [illegible] $\| \cdot \|_{H(\cdot)}$ [illegible] $\| \cdot \|_{L(\cdot)}$ [illegible] $\| \cdot \|_{H^s(\cdot)}$ [illegible]

2.step: [illegible] $\in \mathbb{N}_0$ [illegible]

$$\| \cdot \|_{L^\infty(B)} \ldots \quad \forall \in P_p$$

[illegible] B_j [illegible] $\bigcup_{j=1}^{M} B_j$ [illegible] $\mathbb{R}^d$ [illegible] $\| \cdot \|_{H(\cdot)}$ [illegible] $\bigcup_{j=1}^{M} B_j$ [illegible] $\in P_p$ [illegible]

$$\| \cdot \|_{L(B)} \ldots \| \cdot \|_{L(B)} \ldots \| \cdot \|_{H(B)} \ldots^{d/} \| \cdot \|_{L^\infty(B)}$$

1 Approximation Theory

Theorem 1 *Let* $\mathbb{R}^{\mathbf{d}}$ *be Lipschitz dom in nd et* **f** $_{\mathbf{i}}$**j** − **g** *be co ection of* $^{1;}$) *functions. Set* $_{\mathbf{i}}$ − pp $_{\mathbf{i}}$) , $_{\mathbf{i}}$ − $_{\mathbf{i}}$ *nd ssume*

$$\mathbf{k} \;_{\mathbf{i}}\mathbf{k}_{\mathbf{L}^\infty} \qquad\qquad \mathbf{k}\,\mathbf{r}\;_{\mathbf{i}}\mathbf{k}_{\mathbf{L}^\infty} \quad \frac{\mathbf{G}}{\mathbf{i}} \quad -$$

$$\sum_{\mathbf{i}=1}^{\mathbf{N}} \;_{\mathbf{i}} \quad on \;, \qquad \text{p. r } \mathbf{f}\;\mathbf{2}\;\mathbb{N}\mathbf{j}\;\mathbf{2}\;_{\mathbf{i}}\mathbf{g}$$

Assume th t e ch $_{\mathbf{i}}$, − , *is Lipschitz dom in s e .*
For e ch **2 f** **g** *et* $_{\mathbf{i}}$ ^{1} $_{\mathbf{i}}$) *be gi en nd set*

$$- \sum_{\mathbf{i}=1}^{\mathbf{N}} \;_{\mathbf{i}}\;_{\mathbf{i}} - \left(\sum_{\mathbf{i}=1}^{\mathbf{N}} \;_{\mathbf{i}}\;_{\mathbf{i}}\mathbf{j}\;_{\mathbf{i}}\mathbf{2}\;_{\mathbf{i}} \right)$$

Then ...).
Assu...

1.43994 Td (2)Tj /R28 9.96264 Tf 9.4752

G

V

$\sum_{i=1}^{N}$

$$\int \quad \int \quad \sum_{i=1}^{N} \quad \sum_{i=1}^{N}$$

f j 2 g 2 $\sum_{j=1}^{M}$ $\sum_{j=1}^{M}$ $j=1$ M

$$\sum_{i=1}^{N} \qquad \sum_{i=1}^{N}$$

$$\sum_{i=1}^{N}$$

$$\sum_{i=1}^{N} \qquad \sum_{i=1}^{N}$$

$$\sum_{i=1}^{N}$$

Remark 4.2. $k;q$ 1 2 $k;$ k k k 1 ■

A

Example 4.3. L T $\mathbb{R}^d$. L $f_i j$ g T $f_i j$ g $1;1$ T $f_i j$ g ■

Remark 4.4. N $f_i j$ g i A

[redacted]

$$\inf_{v \in V_N} \| u - v \|_{H^1(\Omega)} \le C N^{-\min\{p,k-1\}} \| u \|_{H^k(\Omega)}$$

■

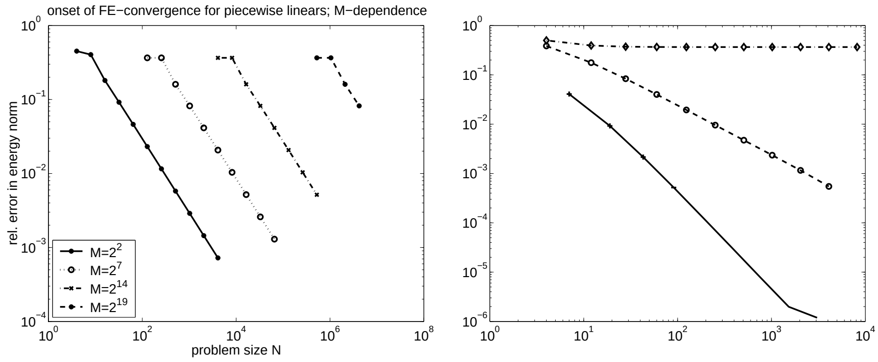
Exercise 4.8.

$$\inf_{v \in V} \| u - v \|_{H^1(\Omega)} \le C \, p^{-p}$$

■

5 Examples of operator adapted approximation spaces

1 A one dimensional example


onset of FE−convergence for piecewise linears; M−dependence
rel. error in energy norm
problem size N
M=2^2
M=2^7
M=2^14
M=2^19

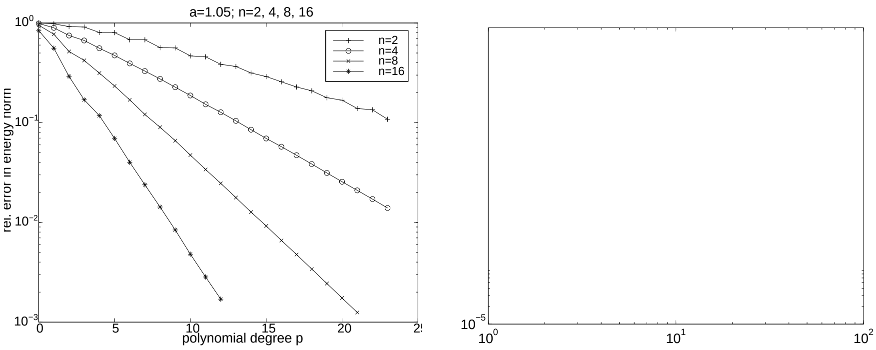
$\int_{\mathbf{x}_0}^{\mathbf{x}}$

j) j) k k_{L^∞}

j) j

[illegible] *independent* [illegible]

2 Laplace's Equation


a=1.05; n=2, 4, 8, 16
n=2
n=4
n=8
n=16
rel. error in energy norm
polynomial degree p

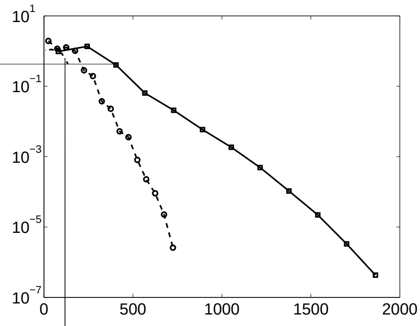
pne es

Gener ized rmonic po ynom s

Theorem 1 *Let* $\mathbb{R}$ *be simp y connected dom in, be com ct subset. Let so e (5.6). Then there exist , such th t for* $2\ \mathbb{N}$, :

Proof.

Theorem 1 *Let* $\mathbb{R}$ *be st r-sh ped ith respect to . Let tisfy n exterior cone condition ith ng e . Let* 2 *, , so e (5.6). Then there exists such th t*


10^1
10^−1
10^−3
10^−5
10^−7
0
500
1000
1500
2000

Theorem 1 *Let* $\mathbb{R}$ *be st r-sh ped ith respect to . Let t-isfy n exterior cone condition ith ng e . Let* $\in \mathbb{N}$, $\in$ *nd s-sume th t the disp cement fie d* $\in$ *tisfies the homogeneous sticity equ tions (5.13). Then the function* $\mathbf{u}$ *i n be pproxim ted from* V_p^{elast} *such th t*

$$\inf_{\mathbf{u}_{ap} \in V_p^{\text{elast}}} \|\mathbf{u} - \mathbf{u}_{ap}\|_{H} \ldots \|\mathbf{u}\|_{H^{m,s}}$$

Proof.

Remark 5.18.

Further examples

ye2078df

[illegible]

for some numbers [illegible] $ij \in \mathbb{R}$ *and* [illegible] $_0 \in$ [illegible].

Proof. [illegible]

Exercise 6.2. [illegible]

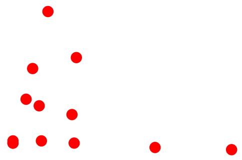
$$\min\{p;k\}$$

■

Exercise 6.3. [illegible]

■

[illegible]



Rem rk 6.5. [illegible]

$2\ P_p$ [illegible] ■

[illegible] N [illegible]

7 Enforcement of essential boundary conditions

Conforming methods:

Non-conforming methods:

1 Conforming methods

A simple approach [illegible]

$B = \{ f_i \mid$ [illegible] $\}$ [illegible] $i \in$ [illegible]

[illegible]

$$N_{;0} = \text{span}\{ f_i \mid \ldots \}$$

[illegible] $N_{;0}$ [illegible]

Exercise 7.1. Let $N = \text{span}\{ f_i \mid$ [illegible] $\}$ [illegible]

Combination with the classical FEM A [illegible]

[illegible]

Example 7.5. Let [illegible] $\mathbb{R}$ [illegible] Let N [illegible]

[illegible]

Let h [illegible] Let T [illegible]

[illegible]

2 Non conforming methods: Lagrange Multiplier methods and collocation techniques

[illegible]

Exercise 7.8. [illegible] ∎

[illegible]

Non conforming methods: penalty method

[illegible]

$$Z$$

[illegible]

Theorem (penalty method). *Let* $\mathbb{R}^d$ *be a Lipschitz domain. Let . Assume* $u \in H^k(\Omega)$ *is the solution of (7.1). Let* $\in H^{k-1}(\Omega)$ *some* [illegible] *on* [illegible] *on* [illegible]*.*

Assume that the approximation space $V_N \subset H^1(\Omega)$ *satisfies:*

[illegible]

Then there holds for [illegible] *independent of* [illegible] *and* [illegible]

[illegible]

Setting — *ith the optim* *ue* — $\frac{k-1}{}$ *gives*

$$\| \quad {}_N \|_H \quad -$$

Remark 7.10. ... 2 $^{k-1}$) ... ■

Proof of Theorem 7.9. ... L ... L ... p) ... ^{1}) ... k k —) ... k $\|_H$ ...

$$\| \quad {}_N \| - \quad \| \quad \| \quad v, v_N$$

... — ...

...) — ...)

Remark 7.11. [illegible]

Remark 7.16.

Lemma 1 *Set*

Exercise 7.19. A 2

$\| \; \|_{L} \quad \| \; \|_{H} \quad \| \; \|_{H^k} \quad \| \; \|_{H^k}$

■

Remark 7.20. N N

■

A Results from Analysis

Theorem A 1 (universal extension operator). *Let* $\mathbb{R}^d$ *be Lipschitz domain. Then there exists linear operator* $\to \mathbb{R}^d$ *with the following properties:*

(i) *for* 2.

(ii) *For each* $2\ \mathbb{N}_0$, $2 \quad 1$, *there exists such that* $\| \; \|_{W^{k;p}} \quad \| \; \|_{W^{k;p}}$ *for* $2^{k;p}$.

Proof. □

Theorem A 2 (multiplicative trace theorem). *Let* $\mathbb{R}^d$ *be Lipschitz domain,* 2. *Then there exists constant such that for* 2^{s} *the trace satisfies*

$$\| \; \|_{L} \quad \| \; \|_{L}^{1-1=} \; \| \; \|_{H^s}^{1=}$$

Proof. 2 A A

$$\| \; \|_{L} \quad \| \; \|_{B} \qquad \text{A}$$

$1= \quad ;1 \quad 1 \quad 1= \; ;1 \quad 2 \quad 1= \; ;1$

$s \quad 1= \; ;1 \quad s \quad ;1$

A $\| \; \|_{B} \quad \| \; \|_{L}^{1-} \; \| \; \|_{H^s}$

□

Here, the notation) represents the function 7!) − f g. *The constant* p;q;k *depends only on* 2 $\mathbb{N}_0$, 2 1), *and* . *The bound (B.2) also holds for* − 1 *if and are restricted to integer values* , 2 $\mathbb{N}_0$.

f 2 1) *and or if* − *and* , *then additionally*

$$k \quad p \; k_{L_\infty(B)} \quad p;q;k \; ^{\min\{p \; 1;k\} - d=q} \, k \; k_{W^{k;q}(B)}$$

here p;q;k *depends only on* , , *and* .

Proof. [illegible]

1. step: L [illegible] 1) ! [illegible] x)

C Approximation with adapted function systems

C.1 The theory of Bergman and Vekua

$\mathbb{R}$

Lemma C.1 *Let $\mathbb{C}$ be simp y connected Lipschitz dom in. Fix ${}_0 \in$. Let H f j ho omorphic on nd $_0$) g. Then there exists in r p ith the fo o ing properties:*

1. *) so es (C.1) for every $\in$ H.*
2. *For every so ution of (C.1) there exists unique $\in$ H such th t) .*
3. *for $\in$ H nd .*
4. *f $\in$), , so es (C.1), then the corresponding $^{-1}$) $\in$ H is i e ise in) nd*

n the st t o estim tes, the const nt depends on , nd the different oper tor.

Proof. ∎

Rem rk C.2. ∎

2. step: [illegible]

Lemma C *Let* $\mathbb{R}$ *be st r-sh ped ith respect to* $_0$)*. Let the disp cement fie d* $\mathbf{u}$ — i 2 ^{k}) *for some* 2 $\mathbb{N}$*. Let* $_0$ 2 *. Let* , *be the ho omorphic functions ppe ring in the represent tion formu (5.14), hich re unique y determined by stipu ting* $_0$) — *. Then*

$$\| \ \|_{H^k(\ )} \ \ \| \ \|_{H^{k-}(\ )} \ \ \|u\|_{H^k(\ )}$$

here depends on y on the L mé const nts, upper bounds on , nd o er bounds on .

Proof. [illegible] 2 [illegible] $\| \ \|_{L}$ [illegible] $\|u\|_{H}$ [illegible] 2 [illegible] $\|u\|_{H}$ [illegible] $_0$) — [illegible] $\| \ \|_{L}$ [illegible] $\| \ \|_{L}$ [illegible] $\mathbf{u}$

Then m *is defined on* $\frac{1}{1-\varepsilon}$ *and*

k mkL ... kukH m ...

k mkH ... −1kukH m ...

k ... mkL ... k ... mkH ... m skukH m, s ...

Proof. [illegible]

ap 2 Hp

k 1 apkW j;∞) −pk 1kL ‖Int‖L)

k 1 apkW j;∞) −pk 1kL ‖Int‖L)

h − f) jj j − g ℂ n 1) ! ℂ n

1) − 1 1)

h $\frac{1}{1-"=}$

2 f g

ku ap ap ap) kH j

m s−

Lemma C *Let* ℂ *be st r-sh ped ith respect to nd ssume th t*) *. Then for* **2** ¹) *ho omorphic on e e*

$$\mathbf{k} \;) \mathbf{k_L} \quad \mathbf{p}_{-} \quad - \quad \overline{\qquad} \quad \mathbf{k} \; \mathbf{k_L} \quad)$$

Proof.

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