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Approximate iterative methods for variational data assimilation

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with $x \in \mathbb{R}^n$ [3]. We can write the 4D-Var objective function (1) in this form by putting $f(x) = C^{-1/2}\hat{d}$, where

$$\hat{d}(x_0) = -\begin{pmatrix} x_0 - x^b \\ H_0[x_0] - y_0^o \\ \vdots \\ H_n[x_n] - y_n^o \end{pmatrix}; \quad C^{-1} = \begin{pmatrix} B_0^{-1} & 0 \\ 0 & R^{-1} \end{pmatrix} \quad \text{with} \quad R = \operatorname{diag}\{R_i\}.$$

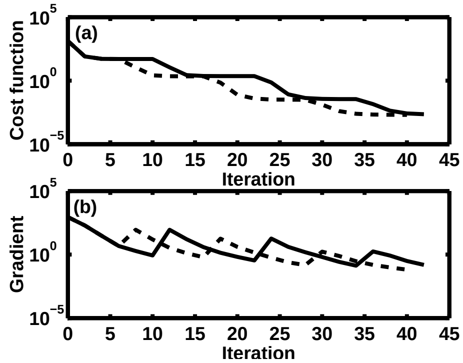

Figure 1: Comparison of convergence of (a) objective function and (b) its gradient for a constant convergence criterion (solid line) and a variable criterion (dashed line).

with $D/Dt = \partial/\partial t + u\partial/\partial x$. In these equations $\bar{h} = \bar{h}(x)$ is the height of the bottom orography, u is the velocity of the fluid and $\phi = gh$ is the geopotential, where g is the gravitational constant and $h > 0$ the depth of the fluid above the orography. The problem is defined on the domain $x \in [0, L]$, with periodic boundary conditions, and we let $t \in [0, T]$.

The model is discretized using a two-time-level semi-implicit semi-Lagrangian scheme. Further details of the numerics can be found in [6]. We use a periodic domain of 1000 grid points, with a spacing $\Delta x = 0.01\ m$, so that $x \in [0\ m, 10\ m]$. The model time step is taken to be $9.2 \times 10^{-3}\ s$ and we consider an assimilation over a window of 100 time steps.

In order to test the assimilation we run 'identical twin' experiments, in which observations are generated from a run of the model defined to be the truth. These observations are then assimilated using 4D-Var, starting from an incorrect prior estimate. The inner minimization is performed using a conjugate gradient method and is considered to

Again it is possible to provide a theoretical understanding of these results by an analysis of the (PGN) method. The method can be considered as a way of solving the approximate normal equations

$$\tilde{J}(\tilde{x}^*)^T f(\tilde{x}^*) = 0. \tag{14}$$

Then based on the work of [7] and [8] we can prove the following theorem:

Theorem 2 *Let the first derivative of* $\tilde{J}(x)^T f(\tilde{x})$ *be written*

$$\tilde{J}(x)^T J(x) + \tilde{Q}(x), \tag{15}$$

where $\tilde{Q}(x)$ *represents second order terms arising from the derivative of* $\tilde{J}(x)$*. Suppose that on each iteration k*

[4] Ortega JM, Rheinboldt WC. *Iterative Solution of Nonlinear Equations in Several Variables*