The State Estimation of Flow Demands in Gas Networks from Sparse Pressure Telemetry

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Abstract

In this paper we present novel observer based techniques for the estimation of flow demands in gas networks from sparse pressure telemetry. We explore a *completely observable* model constructed by incorporating difference equations that assume the flow demands are steady. Since the flow demands usually vary slowly with time, this is a reasonable approximation. Two techniques for constructing *robust* observers are employed: robust eigenstructure assignment and singular value assignment. These techniques help to reduce the effects of the system approximation. Modelling error may be further reduced by making use of known profiles for the flow demands.

The theory is extended to deal successfully with the problem of measurement bias. The pressure measurements available are subject to constant biases which degrade the flow demand estimates, and such biases need to be estimated. This is achieved by constructing a further model variation that incorporates the biases into an augmented state vector, but now includes information about the flow demand profiles in a new form.

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, , [1], [2] system matrices
system measurement matrix
_( ), _( ) system state and measurement vectors
, observer feedback matrices
_( ), _( ) observer state vector and error
Λ set of observer system eigenvalues
individual observer system eigenvalues
_ , _ left and right eigenvectors of observer system
( ) condition number of observer ﬀerT)1Bgenvalues

1

_1
_2
_3
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For any gas network, it is desirable to have a reasonable estimate of the demand flows. However, flow meters are much more expensive than pressure sensors to install, and so it would be economical to be able to estimate the flow demands from pressure measurements alone. In this paper novel observer based techniques for achieving this are presented. The gas networks considered are linear and consist of a number of pipe sections with a gas source at the upstream end and flow demands at pipe junctions and at the downstream end. For example, a three pipe network would be as in Fig.1. We assume the only measurements of the real gas network available are discrete pressure measurements at all

1 2

1 2

2.2 Design 2 : The Dynamic Observer with Feedback at the Present Time Level

The second observer design takes the form

$$(\quad)\_(\; +1) = (\quad)\_(\;) + \; ^1\_(\; +1) + \; ^2\_(\;) \quad \_(\; +1) + \; \_(\;) \qquad (9)$$

where the matrices and are to be chosen. The term in represents feedback at the present time level, and the term in represents the familiar feedback at the previous time level. If we define the error, _( ), between the two systems (1) and (9) at time level to be as in equation (5), then subtracting equation (9) from equation (1) gives

$$(\quad)\_(\; +1) = (\quad)\_(\;) \qquad (10)$$

If can be chosen such that ( ) is nonsingular, then its inverse can be calculated

1

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1

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1 1

2 1 2

2 1

1

1

2

3.1 The Base Model

3.2 Observable Models

New models, which we term models, can be constructed assuming the flow demands are fairly steady, i.e.

$$ {}_{+1} \quad = $$

These scalar equations may be written as the system

$$ \_ + \_(\; + 1) \; = \; \_ + \_(\;) $$

where _ is a vector containing the steady flow demands, about which the gas network model is linearised. Hence, we derive the following trivial difference equations

$$ \_(\; + 1) = \_(\;) \tag{14} $$

To form a model, we start from a base model and move the variables, _( ), from the input vector to the state vector. We then introduce new trivial difference equations of the form (14) into the new system. The new + dimensional system has the form

$$ \begin{matrix} {}^{1} & {}^{2} & {}^{1}_{2} \\ 0 & 0 & \end{matrix} \quad \begin{matrix} {}_{\_1}(\; + 1) \\ {}_{\_2}(\; + 1) \\ \_(\; + 1) \end{matrix} \; = \; \begin{matrix} {}^{1} & {}^{2} & {}^{2}_{2} \\ 0 & 0 & \end{matrix} \quad \begin{matrix} {}_{\_1}(\;) \\ {}_{\_2}(\;) \\ \_(\;) \end{matrix} $$

$$ + \; \begin{matrix} {}^{1} \\ {}^{1} \\ 0 \end{matrix} \; {}_{\_3}(\; + 1) + \; \begin{matrix} {}^{2} \\ {}^{1} \\ 0 \end{matrix} \; {}_{\_3}(\;) \tag{15} $$

The only input required for the model is ${}_{\_3}(\;)$ which is assumed known. The measurements of ${}_{\_2}(\;)$ are not needed as inputs to the model, and are in fact measurements of its state variables

$$ \_ \qquad {}_{\_1} \quad {}_{\_2} \quad \_ \qquad {}_{\_2} $$

modelling error. If the flow demands are changing, although not too rapidly, the observer may still track the state of the gas network fairly well. In practice, the observer state estimates are expected to be fairly accurate since the flow demands in gas networks change only slowly throughout the day.

In fact, the profiles of the flow demands may be fairly well known from other measured demands that change thoughout the day with similar patterns of gas consumption. More accurate models may be constructed using specific information available about the flow demand profiles with time. At each timestep, , the vector _( ) is estimated where

$$\_(\;+1) = \_(\;) + \_(\;) \tag{17}$$

The models now incorporate trivial difference equations of the form (17) for the flow demands, where the vector, _( ), is added to the right hand side of the system. Note that only the of the flow demand profiles are needed and not their precise values. Adding _( ) to the right hand side of the system does not alter the observability of the model.

3.3 Experiments and Discussion

A standard model of a linear three pipe network is run with the upstream pressure perturbation, junction flow demand perturbations, and downstream flow demand perturbation specified as boundary inputs to the system. This model is identical to the model upon which all model based observers are constructed. Model parameters for all experiments are given in appendix A.

After the model is run for a short while, the pressures at the upstream end and at where the vector _( ) contains the error terms.

The dynamic observers are built around the original system (15). If we define the error between (18) and the observer (9) at time level to be _( ) = _( ) _( ), then subtracting the observer system (9) from (18) gives

$$(\quad)\_(\quad + 1) = (\quad)\_(\quad) + \_(\quad) \tag{19}$$

where = 0 for Design 1 observers, and where _( ) acts as a forcing term on the errors.

For Design 2 observers, the matrix is chosen to minimise the 2-norm of ( ) ^{1}, and this matrix is implicitly multiplied into the forcing term, _( ), thus reducing its effects. In Figs. 2 and 3, the 2-norm of the matrix ^{1} is 1 50, while the 2-norm of ( ) ^{1} is 0 56 and this is believed to explain the significant improvement in the accuracy of the state estimate when feedback is included at the present time-level.

It is also found that the state estimates contain less error when smaller eigenvalues are assigned to the observer systems (Stringer, 1993). If the eigenvalues of the observer system are small, then so are the eigenvalues of the error system (19), and hence the errors damp down more quickly.

_( )

When the weightings, _( ), are included in the model, the state estimates can converge perfectly, and assigning smaller system eigenvalues to the observers gives faster convergence. However, in practice, the _( ) would probably be slightly innacurate, giving some small error in the observer state estimate.

It may be the case that the pressure measurements at the sites of flow demand are subject to a constant bias, i.e.

$$\_(\quad) = \_{2}(\quad) + \_(\quad) \tag{20}$$

where _( ) is a dimensional vector of constant measurement biases, which are assumed to obey

$$\_(\quad + 1) = \_(\quad) \tag{21}$$

The asymptotic estimates of flow demands from models and observers presented so far can be shown to be sensitive to pressure measurement biases (see Fig.4). This is a serious problem for flow demand estimation and the adverse effects of these pressure measurement biases need to be eliminated. However, in (Stringer, 1993), it is shown that when running a model, or either type of observer based upon a model, any time series of pressure inputs and measurements which have been corrupted by any set of constant pressure sensor biases at the sites of flow demand, are perfectly consistent with some time series of or system states with no pressure input or measurement biases, which will be the actual state estimates given (asymptotically, in the case of a dynamic observer).

For a proof of this theorem, see appendix B.

When the Hautus condition holds, we can find a feedback matrix ( ) to assign observer eigenvalues arbitrarily at that timestep . Since it can be shown that the base gas network model is completely observable for 0, and has no eigenvalue equal to one for 1 2 1, the Hautus condition may fail to hold for a model with 1 2 1 only for a few specific values of the coefficients . At these particular timesteps we can run the simple model (22) without the observer feedback terms. As the model based observer steps forward in time, it is intended that, the observer state estimate should converge asymptotically to the true state of the gas network.

4.1 Experiments and Discussion

In these experiments, the pressure measurements at the three flow demand sites, , and , are corrupted by constant biases of 1 bar, 1 bar and 1 bar respectively, before being fed into the and model based observers. Graphical results are given for the following experiments.

Fig.4 Model Based Observer Design 2 with = 1 0.

Fig.5 Model Based Observer Design 1 with = 1 0

The flow demand estimates of both the Design 1 and Design 2 observers based on models are completely swamped by the error due to pressure measurement bias, which is demonstrated by Fig.4.

For = 1 0, the Design 1 observer converges successfully giving accurate flow demand estimates, as shown in Fig.5. However, for any value of , it has been found that the models are much more sensitive to modelling error and measurement noise (Stringer, 1993).

worn

modffwT)HH1Λ4ffisT)HH1Λ4gen2FF1Λ'ffeT'ΛffesgtratedT)HH1ΛusivingntnffamT2Λ'ffeT'ΛFffnTffi

nand)F3FΛHffwith)F3FΛ(StviplT'Λicalffi'1Λdieccb,ff.TffiF+4Λ+fftheT'flowdemasffwT'vsrvy modsff

trffyT)Fffi'ΛF1ffoT)ffi'ΛffiffdelT)Fffi'ff2ffv3T+Λariaditi+ff,T)Fffi'ff(5.H323'Λ)ff+FΛes,Tm'H3Λ+ffwhiF

It should be emphasised that all the experiments in this paper have been performed using *simulated* pressure data. In fact, although experimental results have not been presented here, all flow estimation techniques are badly affected in practice by measurement noise due to the underlying sensitivity of the flow estimates. This problem has been further explored in (Stringer, 1993) with further model variations that include a *total* flow variable that is the sum of all the individual flow demands. The flow estimates of such models are much less sensitive to measurement noise. Two smoothing methods presented in (Stringer, 1993), also significantly improve the flow estimates adversely affected by measurement noise.

[1] 1985. Introduction to Modern Control Theory (2nd dition). Clarendon Press, Oxford.
[2] 1992. 'Regularisation of

7.1 Construction of Base Model

and Ω and Γ are known functions of

$$\Omega = \ _2\ _3(1 \quad)^2 \qquad \Gamma = \quad (1 \quad)$$

and $_3 = 2\ (\ _1 \quad ^1)$.

For each pipe section, equation (26) is approximated by the finite difference equation

$$\begin{aligned} {}^{+1} \quad &= \Omega \ (\Gamma \ _1 \quad _{1\ +1} \quad 2\Gamma \quad _{+1} + \Gamma_{+1} \ _{+1\ +1}) \\ &+ (1 \quad)\Omega \ (\Gamma \ _1 \quad _1 \quad 2\Gamma \quad + \Gamma_{+1} \ _{+1} \ ) \end{aligned} \qquad (29)$$

for $0 \quad 1$, where $= \quad (\quad)^2$. For a downstream *flow* boundary condition given at node $=$ we approximate equation (27) by

$$_3(\Gamma_{+1} \ _{+1} \quad \Gamma \ _1 \quad _1 \ ) = 2 \qquad (30)$$

liminating $\Gamma_{+1} \ _{+1}$ and $\Gamma_{+1} \ _{+1\ +1}$ between equations (29) and (30) then gives at node the difference equation

$$\begin{aligned} &2\ \Omega \ \Gamma \ _1 \quad _{1\ +1} + (1 + 2\ \Omega \ \Gamma \ ) \quad _{+1} + (2\ \Omega \quad _3) \quad _{+1} = \\ &2(1 \quad)\Omega \ \Gamma \ _1 \quad _1 \ + (1 \quad 2(1 \quad)\Omega \ \Gamma \ ) \quad (2(1 \quad)\Omega \quad _3) \end{aligned} \qquad (31)$$

An equivalent difference equation may be derived for an upstream flow boundary condition.

For an outflow, $^{+1}$, at the junction of two pipes, denoted by superscripts and $+1$ respectively, the pipe end flow boundary equations may be used to derive

$$^{+1}\ _3 \quad _1 \quad _{1\ +1} \quad ^{+1}\ _3 \quad ^{+1} \quad ^{+1}\ _3^{+1} \quad ^{+1} \quad ^{+1} \quad ^{+1}_{+1}$$

$$^{+1}\ _3^{+1}\ _1^{+1} \quad ^{+1}\ _3^{+111} \quad 3{+}11 \quad {+1}$$

where we have defined

$$\Phi \quad ^{+1} = \left(\frac{_3}{2 \qquad \Omega} + \frac{_3^{+1}}{2 \quad ^{+1} \quad ^{+1}\Omega_0^{+1}}\right)^{1}$$

and where, because $_0^{+1}$ and $\Gamma \quad \Gamma_0^{+1}$, at the junction we have denoted the pressure perturbation by $^{+1} \qquad _0^{+1}$, and denoted Γ by $\Gamma \ ^{+1} \ \Gamma \quad \Gamma_0^{+1}$. This equation is the 'connectivity equation', which is used at the junction nodes.

7.2 Model Parameters for Experiments

The values of the model parameters for all pipes used in our experiments are given by

$$_1 = \frac{\quad}{_0} \qquad _2 = \frac{10^6}{\quad}$$

where $= 7000$ is the length of the pipe, $= 1\ 0 \quad ^2$ is the cross-sectional area of the pipe, $= 1\ 854$ is the flow exponent, $= 0\ 105$ is the friction coefficient, $= 300^0$ is the temperature of the gas, $= 288\ 7056^0$ is the temperature at standard conditions, $= 1\ 0138$ is the pressure at standard conditions, $= 0\ 9978$ is the compressibility at standard conditions, $_0 = 1\ 0$ is the compressibility at reference conditions, and $=$ 0 00205946. In all numerical experiments, the timestep used for the models was 0 1666665 hours (= 10), and each pipe had ten computational nodes. The state variables of the flow estimation models and dynamic observers were initially set to zero.

8 Appendix B - Proofs of Theorems

8.1 Proof of Theorem 3.3

¿From the Hautus condition we have that the γ system is observable if and only if for all $\mu \in \mathbf{C}$

$$[(A_1 - \mu E_1) \quad (A_2 - \mu E_2) \quad (B_2^2 - \mu(-B_2^1))] \begin{bmatrix} \underline{v}_{n-g} \\ \underline{v}_g^1 \\ \underline{v}_g^2 \end{bmatrix} = \underline{0} \tag{33}$$

$$(1 - \mu)\underline{v}_g^2 = \underline{0} \tag{34}$$

$$\underline{v}_g^1 = \underline{0} \tag{35}$$

$$\Longleftrightarrow$$

$$\underline{v}_{n-g} = \underline{0}\ ,\ \underline{v}_g^1 = \underline{0}\ ,\ \underline{v}_g^2 = \underline{0} \tag{36}$$

where $\underline{v}_{n-g} \in \mathbf{R}^{n-g}$ and $\underline{v}_g^i \in \mathbf{R}^g$ for i=1,2.

Consider the case of $\mu \neq 1$. quation (34) implies $\underline{v}_g^2 = \underline{0}$. Substituting $\underline{v}_g^2 = \underline{0}$ into equation (33) gives

$$[(A_1 - \mu E_1) \quad (A_2 - \mu E_2)] \begin{bmatrix} \underline{v}_{n-g} \\ \underline{v}_g^1 \end{bmatrix} = \underline{0}. \tag{37}$$

If the α system is completely observable, then the Hautus condition holds for the α system, and equations (35) and (37) imply $\underline{v}_{n-g} = \underline{0}$ and $\underline{v}_g^1 = \underline{0}$.

Consider the case of $\mu = 1$. Removing $\underline{v}_g^1$ from system (33) gives the system

$$[(A_1 - E_1) \quad (B_2^2 - (-B_2^1))] \begin{bmatrix} \underline{v}_{n-g} \\ \underline{v}_g^2 \end{bmatrix} = \underline{0}. \tag{38}$$

By inspection, if 1 is not an eigenvalue of the corresponding β system, equation (38) implies $\underline{v}_{n-g} = \underline{0}$ and $\underline{v}_g^2 = \underline{0}$. □

8.2 Proof of Theorem 4.1

The Hautus condition holds for the δ system if and only if for all $\mu \in \mathbf{C}$

$$[(A_1 - \mu E_1) \quad (A_2 - \mu E_2) \quad (B_2^2 - \mu(-B_2^1))] \begin{bmatrix} \underline{v}_{n-g} \\ \underline{v}_g^1 \\ \underline{v}_g^2 \end{bmatrix} = \underline{0} \tag{39}$$

$$(W(k) - \mu I)\underline{v}_g^2 = \underline{0} \tag{40}$$

$$(1 - \mu)\underline{v}_g^3 = \underline{0} \tag{41}$$

$$\underline{v}_g^1 + \underline{v}_g^3 = \underline{0} \tag{42}$$

$$\Longleftrightarrow$$

$$\underline{v}_{n-g} = \underline{0}\ ,\ \underline{v}_g^1 = \underline{0}\ ,\ \underline{v}_g^2 = \underline{0}\ ,\ \underline{v}_g^3 = \underline{0} \tag{43}$$

where $\underline{v}_{n-g} \in \mathbf{R}^{n-g}$ and $\underline{v}_g^i \in \mathbf{R}^g$ for i=1,2,3.

We firstly consider the case where $\mu \neq 1$.

quation (41) implies $\_^3 = \underline{0}$. Substituting $\_^3 = \underline{0}$ into equation (42) gives $\_^1 = \underline{0}$.

If $=$ for all flow demands, equation (40) gives $\_^2 = \underline{0}$. Substituting $\_^2 = \underline{0}$ into equation (39) gives

$$[(\quad _1 \quad _1) \quad (\quad _2 \quad _2)] \quad \bar{\underline{\ }}_1 \quad = \underline{0} \tag{44}$$

If the system is completely observable, then the Hautus condition holds for the system, and $\_^1 = \underline{0}$ together with equation (44) imply $\_ \quad = \underline{0}$.

If $=$ for any flow demands, removing $\_^1$ from system (39) gives

$$[(\quad _1 \quad _1) \quad (\quad ^2_2 \quad (\quad ^1_2))] \quad \bar{\underline{\ }}_2 \quad = \underline{0} \tag{45}$$

$\_ \quad \_ \quad \_^2 \quad \_$

$\_^2 \quad \_$

$\_^2 \quad \_$

$_1 \quad _1 \quad _2 \quad _2 \quad \bar{\underline{\ }}_1 \quad \_$

$\_ \quad \_^1 \quad \_^1 \quad \_ \quad \_^3 \quad \_$

List of Captions for Figures